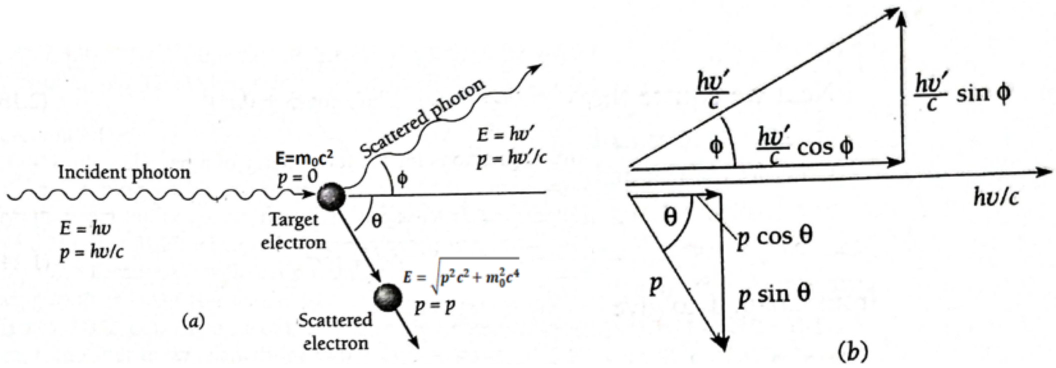
Compton effect:

According to quantum theory of light, photons behave like particles. Therefore, collision of a photon and an electron can be considered.



Let an X-ray photon strikes an electron which is at rest. The photon is scattered at an angle ϕ from its original direction of motion where the electron receives an impulse and is recoiled at an angle θ with the original direction of motion of the photon. As a result of the collision the photon will lose an amount of energy and the same amount of energy will be gained by the electron as kinetic energy, although after the collision scattered photon will be different from the origin photon. If the initial photon has frequency ν associated with it, the scattered photon has lower frequency ν'. Kinetic energy gained by the electron is given by

$\mathrm{KE} = h\nu - h\nu'$…………………………………(1)

Energy of a particle of a relativistic particle is given by

$E = \sqrt{p^2c^2 + m_0{}^2c^4}$ ………………………………(2)

where m_0 is the rest mass, c the velocity and p the momentum of the particle.

Since the rest mass of a photon is zero, its energy is given by

$E = pc$ ……………………………….(3)

Energy of a photon is also given by $E = h\nu$. Therefore, its momentum is given by

$p = \frac{E}{c} = \frac{h\nu}{c}$ ………………………………(4)

Hence, the momentum of the initial photon is $\frac{h\nu}{c}$, while that of the scattered photon is $\frac{h\nu'}{c}$. Initial and the final momentum of the electron are 0 and p respectively.

Since momentum also must be conserved in the collision, we equate the components of the momenta in the direction of motion of the original photon and another direction which is perpendicular to it and also in the same plane containing the original direction of photon and the direction of recoil of the electron. In the origin photon direction we have,

$$\frac{h\nu}{c} = \frac{h\nu'}{c}Cos\phi + pCos\theta \text{ ....(5)}$$

and in the perpendicular direction,

$$0 = \frac{h\nu'}{c}Sin\phi - pSin\theta \text{ ....(6)}$$

From equations (5) and (6) we have,

$$pcCos\theta = h\nu - h\nu' Cos\phi \text{ ....(7)}$$

$$pcSin\theta = h\nu' Sin\phi \text{ ....(8)}$$

By squaring these equations and adding them we have,

$$p^2c^2(Cos^2\theta + Sin^2\theta) = (h\nu)^2 - 2(h\nu)(h\nu')Cos\phi + (h\nu')^2(Cos^2\phi + Sin^2\phi)$$

Or, $p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')Cos\phi + (h\nu')^2$ ....(9)

The expressions of energy of a relativistic particle are given by

$$E = m_0c^2 + KE$$

and $E = \sqrt{p^2c^2 + m_0^{\ 2}c^4}$

Comparing these two expressions we have,

$$(m_0c^2 + KE)^2 = p^2c^2 + m_0^{\ 2}c^4$$

Or, $p^2c^2 = KE^2 + 2m_0c^2KE$ ....(10)

Substituting the value of KE from eq. (1) in eq. (10) we have,

$$p^2c^2 = (h\nu - h\nu')^2 + 2m_0c^2(h\nu - h\nu') \text{ ....(11)}$$

Now, comparing equations (9) and (11) we have,

$$(h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2m_0c^2(h\nu - h\nu') = (h\nu)^2 - 2(h\nu)(h\nu')Cos\phi + (h\nu')^2$$

Or, $2m_0c^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1 - Cos\phi)$

Or, $\dfrac{\nu - \nu'}{\nu\nu'} = \dfrac{h}{m_0c^2}(1 - Cos\phi)$

Or, $\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 c^2}(1 - Cos\phi)$

Or, $\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0 c}(1 - Cos\phi)$

Or, $\lambda' - \lambda = \frac{h}{m_0 c}(1 - Cos\phi)$ ....(12)

Here λ and λ' are the wavelengths of the incident and the scattered photons respectively. In 1924, Compton while experimentally studying the scattering of X-rays from carbon, observed that the radiation scattered in a particular direction consisted of X-rays of two different wavelengths. One of these had the same wavelength as the incident radiation, while the wavelength of the other was somewhat longer, so its frequency was lower. The inelastic scattering phenomenon in which the wavelength (or frequency) of the radiation changes, is called the *Compton scattering* or the *Compton effect* after the name of the discoverer. Compton effect provides a strong support for the quantum theory of radiation. Eq. (12) was derived by Compton himself assuming that photons not only have energy, but also has momentum like a material particle.

Eq. (12) gives the change in wavelength for a photon that is scattered through an angle ϕ by a particle of rest mass m_0 . This change is independent of the wavelength λ of the incident photon and depends on the angle through which the photons are scattered. The quantity

$\lambda_C = \frac{h}{m_0 c}$ ....(13)

is called the Compton wavelength of the scattering particle. For an electron

$\lambda_C = 2.426 \text{ x } 10^{-12} m = 0.024 \overset{0}{\text{A}}$

Thus, the Compton shift is given by

$\Delta\lambda = \lambda' - \lambda = \lambda_C (1 - Cos\phi) = 2\lambda_C Sin^2 \frac{\phi}{2}$ ....(13)

The Compton wavelength gives the scale of wavelength change of the incident photon. From eq. (12) and (13) we note that the greatest wavelength change is possible corresponding to $\phi = 180^0$ and the wavelength change is twice the Compton wavelength λ_C. $\lambda_C = 0.024 \overset{0}{\text{A}}$ and the maximum wavelength change possible is $0.048 \overset{0}{\text{A}}$ for an electron and it is even less for other particles due to their larger rest mass. Change of this magnitude and even less are easily observed only in X-rays ($\lambda \approx 1 \overset{0}{\text{A}}$). The shift in wavelength for visible light is less than 0.01% of the initial wavelength, whereas for X-rays it is several percent. The Compton scattering is the chief means by which X-rays lose energy when they pass through the matter.

Note:

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 c^2}(1 - Cos\phi)$$

Or, $\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0 c^2}(1 - Cos\phi) = \frac{1}{\nu} + \frac{h}{m_0 c^2}.2Sin^2\frac{\phi}{2}$

Or, $\frac{1}{\nu'} = \frac{1}{\nu}\left(1 + 2\frac{h\nu}{m_0 c^2}Sin^2\frac{\phi}{2}\right) = \frac{1}{\nu}\left(1 + 2\alpha Sin^2\frac{\phi}{2}\right)$, where $\alpha = \frac{h\nu}{m_0 c^2}$

$$\therefore \nu' = \frac{\nu}{1 + 2\alpha Sin^2\frac{\phi}{2}}$$

Kinetic energy of the recoil electron is given by

$$E_k = h\nu - h\nu' = h\nu\left(1 - \frac{1}{1 + 2\alpha Sin^2\frac{\phi}{2}}\right) = h\nu\frac{2\alpha Sin^2\frac{\phi}{2}}{1 + 2\alpha Sin^2\frac{\phi}{2}}$$

$$\therefore E_k = h\nu\frac{\alpha((1 - Cos\phi))}{1 + \alpha(1 - Cos\phi)}$$