Poisson Distribution

A discrete random variable X is said to follow a Poisson Distribution if its probability mass function is given by P(r) = P(X=r) = $\frac{e^{-\lambda}\lambda^r}{r!}$ r= 0,1,2……..

= 0 elsewhere

$\lambda > 0$ is known as the parameter of the distribution. A random variable X which is follows Poisson distribution is called a Poisson variate and is denoted by x$\sim P(\lambda)$

Obviously,

(i) P(X=r)$\geq 0\ for\ all\ x$

$\sum_{r=0}^{\infty} \mathrm{P(X=r)} = \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{r!} = e^{-\lambda} \sum_{r=0}^{\infty} \frac{\lambda^r}{r!} = e^{-\lambda} e^{\lambda} = 1$

∴This is a valid probability distribution.

Occurrence of Poisson Distribution

In some situations where a random experiment results in two outcomes success or failure, the number of successes only can be observed and not the number of failures. We can observe how many roads accidents occur but we cannot observe how many accidents do not occur. We can observe how many persons die from cancer but we cannot observe how many do not die from cancer. In such situations Binomial distribution cannot be used.

We use Poisson distribution where the following conditions must be satisfied.

(i) $\mathrm{n} \longrightarrow \infty$
(ii) $p \longrightarrow 0$
(iii) np = constant.

Prob1: If the chance of being killed by flood during a year is $\frac{1}{3000}$, use Poisson distribution to calculate probability that out of 3000 persons living in a village, at least one will die in the flood in a given year.

Solu: Let the random variable X corresponding to the number of persons being killed by flood during a year.

Given, p= probability that one person will die in a year due to flood out of 3000 persons = $\frac{1}{3000}$ $\mathrm{and\ n} = 3000$.

$\lambda = np$=3000× $\frac{1}{3000}$ =1

Average number of person being killed by flood during a year P(X=r) = $\frac{e^{-\lambda}\lambda^r}{r!}$ =$\frac{e^{-1}}{r!}$ $(since\ \lambda = 1)$ r= 0,1,2……..

∴ Required probability = P (X≥ 1)= 1− P (X$= 0$)= 1− $\frac{e^{-1}}{0!}$ =1− e^{-1}

Mean and variance of the Poisson Distribution

Th:If X is a Poisson variate with parameter $\lambda(> 0)$, then (i)Mean = E(X)= λ(ii)Var(X) = λ

(i) Mean = E(X)= $\sum_{r=0}^{\infty} rP(X=r) = \sum_{r=0}^{\infty} \frac{re^{-\lambda}\lambda^r}{r!} = \lambda e^{-\lambda} \sum_{r=1}^{\infty} \frac{\lambda^{r-1}}{(r-1)!}$

$= \lambda e^{-\lambda} \sum_{r=0}^{\infty} \frac{\lambda^r}{r!}$ [replacing $(r-1)$ by r] $= \lambda e^{-\lambda} e^{\lambda} = \lambda$

(ii) (ii) E{x(x−1)} = $\sum_{r=0}^{\infty} r(r-1)P(X=r) = \sum_{r=2}^{\infty} r(r-1) \frac{e^{-\lambda}\lambda^r}{r!}$

$= \lambda^2 e^{-\lambda} \sum_{r=2}^{\infty} \frac{\lambda^{r-2}}{(r-2)!} = \lambda^2 e^{-\lambda} \sum_{r=0}^{\infty} \frac{\lambda^r}{r!}$[replacing $(r-2)$ by r] $= \lambda^2 e^{-\lambda} e^{\lambda} = \lambda^2$

Var(X) = E{x(x−1)}−m(m−1) $= \lambda^2 - \lambda(\lambda - 1) = \lambda$

Standard deviation (S. d) $= \sqrt{\lambda}$

Prob: Let X is a Poisson variate with P (1) = P (2). Find (i) Mean and s. d of X, (ii) P(X = 4)

Solu: P (1) = P (2) ⇒ $\lambda e^{-\lambda} = e^{-\lambda} \frac{\lambda^2}{2!} \Rightarrow = \lambda = 2$

Mean = E(X)= λ=2, Var(X) = λ=2, (S. d) $= \sqrt{2}$

$P(X=4) = e^{-\lambda} \frac{\lambda^4}{4!} = e^{-2} \frac{2^4}{4!} = e^{-2} \frac{2}{3}$

Prob:In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Calculate the number of packets containing no defective, one defective and two defective blades in consignment of 10,000 packets. (Given $e^{-0.02}$=0.9802)

Solu: Given p= probability of defective in a single trial =0.002

n = 10. $\lambda = np = 10 \times 0.002 = 0.02$

P(r) = P(X=r) = $\frac{e^{-\lambda}\lambda^r}{r!}$ r= 0,1,2........

N= number of packets in the consignment=10000

(I) probability for no defective= P(0)= $e^{-0.02}$=0.9802

Number of packets with no defective blades =10000× **0.9802**= 9802.

(II) probability for one defective= P (1) = $\lambda e^{-0.02} = 0.9802 \times 0.02 = 0.019604$

Number of packets with no defective blades =10000× **0.019604**= 196.

(III) probability for two defectives= P (2) = $\frac{\lambda^2}{2!} e^{-0.02} = 0.9802 \times \frac{(0.02)^2}{2} = 0.00019604$

Number of packets with no defective blades =10000× **0.00019604**= 2.