CHAPTER-1

1. Distinguish between Frequency and Variable.

Solution: A measurable characteristic is termed as a variable. Examples of variable are height, weight, age, income etc. Frequency of a variable is the number of times it occurs in a series of observations.

2. What are different parts of a statistical table?

Solution: A table consists of four parts: Title, Stub, Caption and Body.

3. Distinguish with examples between 'Census' and 'Sample Survey'.

Solution: During the census f population of a country each individual person residing in that country is examined and the data are collected. This is the example of 'Census' or 'Complete Enumeration.'

In auditing procedures, there is a popular method called test check method, where auditor checks only a small number of entries taken from different parts of the books of accounts. This is an example of 'Sample Survey.'

4. Name the different methods of presentation of statistical data.

Solution: Statistical data may be presented in three different ways: (i) Textual presentation (ii) Tabular presentation, (iii) Graphical presentation.

5. What do you mean by classification of statistical data?

Solution: Classification is the process by which the statistical data are arranged into groups or classes according to their resemblances. There are four types of classification: (i) on qualitative basis (ii) on quantitative basis (iii) on time basis (iv) on geographical basis.

6. Distinguish between Population and sample, by illustrating an example.

Solution: Sugar bags produced in a factory in a particular day may be considered as population whereas, any specific number of bags out of them would become sample.

7. Write down the objectives of classification.

Solution: Objectives of classification are:

(i) It makes the data understandable.
(ii) It reveals the true significance of the data.
(iii) It arranges the data into different group according to their resemblance.
(iv) It facilities comparison among different groups.

(v) During the process of classification unnecessary and irrelevant data may be eliminated.

8. Define statistics with an example.

Solution: Statistics is the science which deals with the method of collecting, compiling presenting, analyzing, and interpreting numerical data collected.

To measure per capita income of the people of a country from the data collected for this purpose is an of example of statistics.

9. State two characteristics of statistics.

Solution:

i. Statistics deals with only numerical data.
ii. Statistics deals with group of items, not with an individual item.

10. Point out the limitations of statistics.

Solution:

i. Statistics cannot deal with qualitative data.
ii. Statistics cannot deal with individual item.
iii. Statistics cannot give exact result.
iv. Statistics cannot be applied to heterogeneous data.

11. Discuss advantages of graphical representation of statistical data.

Solution:

i. Graphical presentation is attractive and appealing to eyes.
ii. It is easily and quickly understood.
iii. Two or more sets of data can be compared.
iv. An ordinary man can interpret the data through this diagram.
v. It may reveal the hidden facts of the data.

12. Distinguish between census and sample survey and discuss their comparative advantages.

Solution: Census and sample survey both are the methods of collecting statistical data. In census all the items in the population are investigated and in sample survey items in a portion of the population are investigated.

Census requires a large number of investigators and it involves much money and time. The data collected by this method are much reliable and much accurate.

Sample survey involves less money, less time, and less labor. The data collected by this method are less reliable and less accurate. In some cases where census method is not feasible, sample survey method is employed.

13. What do you mean by a questionnaire? State the central pointset be observed in drafting a good questionnaire?

Solution: Questionnaire is a proforma consisting of several questions related to statistical enquiry. It is used for collecting primary data. Questionnaire may be sent through mail or, information may be collected directly by personal contact.

For drafting a good questionnaire following points must be observed.

i. Questions should be free from ambiguity.
ii. Questions should not be lengthy.
iii. Questions must be simple.
iv. Question should be objective in nature.
v. Any personal questions should not be set up.

CHAPTER 2

1. Find the median, mode and mean of the following numbers

7, 4,10,15,7,3,5,2,9,12

Solution: Arranging the numbers in ascending order we have

2, 3, 4, 5, 7, 7, 9, 10, 12, 15

i) Here, number of observations = 10 (even).

By inspection we see that there are two middle terms which are 7 and 7

.: middle most number = $\frac{7+7}{2}$ =7. Hence, median = 7

ii) Here, 7 occurs maximum number of times i.e., 2 times.

.: mode =7

iii) mean $\frac{2+3+4+5+7+7+9+10+12+15}{10} = \frac{74}{10} = 7.4$

2. Find the empirical relation among mean, mode and median.

Solution: mean - mode = 3(mean - median).

3. For a symmetric distribution Q, = 24, Q, = 32, then find the median.

Solution: median = $\frac{Q_1+Q_3}{2} = \frac{24+32}{2} = 28$

4. Find G.M. of 1, 3, 9.

Solution: G.M. = $\sqrt[3]{1 \times 3 \times 9} = \sqrt[3]{27} = 3$

5. If the algebraic sum of the deviations of 25 observations from 45 is -55, find the arithmetic mean.

Solution: A.M = $A + \frac{\Sigma d}{n} = 45 + \frac{-55}{25} = 45 - 2.2 = 42.8$

6. Find the median of 8, 3, 11, 7, 12, 6, 9

Solution: Arranging in ascending order we get 3, 6, 7, 8, 9, 11, 12

By inspection we see that 4th term is the middle term

∴ Middle most value = 4th term = 8 ∴median = 8

7. Find G. M. of 2, 9, 12

Solution: G. M. = $\sqrt[3]{2 * 9 * 12} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} = 2 \times 3 = 6$

8. If mean and median of a distribution are respectively 35 and 33, then find the mode.

Solution: By empirical relation we have, Mean - Mode = 3 (Mean - Median)

or, 35-M = 3(35-33) or, 35-M = 6

or, M=29 ∴Mode = 29

9. Find G.M of 6, 24, 12.

Solution: G.M. = $\sqrt[3]{6 \times 24 \times 12} = (3 \times 2 \times 3 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2)^{\frac{1}{3}} = (3^3 \times 2^6)^{\frac{1}{3}}$

$= 3 \times 2^2 = 3 \times 4 = 12$

10. Find the median 7, 2, 5, 9 6.

Solution: Arranging in ascending order we get 2,5,6,7,9

Median = middle most value = 3rd term =6

11. Find the mode of 5,3,2,7,5,9,3,8,5

Solution: Here, 5 occurs maximum number of times, i.e., 3 times

∴ mode = 5

12. Find the A.M of 14, 16, 19, 25, 21.

Solution: A.M $(\bar{x}) = \frac{14+16+19+25+21}{5} = \frac{95}{5} = 19$

13. Find the mode of 4,3,2,5,3,4,5,3,7,3,2,6

Solution: Arranging the numbers in frequency distribution, we get

x:	2	3	4	5	6	7
y:	2	4	2	2	1	1

Here, 3 occurs with maximum frequency 4 $\therefore$mode = 3

14. Find the median of 38, 86, 68, 32, 80, 48, 70.

Solution: Arranging the numbers in ascending order we get, 32, 38, 48, 68, 70, 80, 86

Here n =7 $\therefore \frac{n+1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4$

$\therefore$ median = 4th term = 68.

15. Find the G.M. of 3, 8, 9.

Solution: G.M. = $\sqrt[3]{3 \times 8 \times 9} = (3 \times 2 \times 2 \times 2 \times 3 \times 3)^{\frac{1}{3}}$

= $(2^3 \times 3^3)^{\frac{1}{3}} = 2 \times 3 = 6$

16. Find the median and mode of the following numbers: 4,10,7,15,7,3,5,3,7.

Solution: Arranging the numbers in ascending order.

We get 3,3,4,5,7,7,7,10,15.

Here n=9 $\therefore \frac{n+1}{2} = \frac{9+1}{2} = \frac{10}{2} = 5$

$\therefore$ median = 5th term = 7

Again, 7 occurs maximum number of times, i.e., 3 times

$\therefore$ Mode = 7.

17. The mean marks of 100 students was found to be 40. Later on it was discovered that a mark 53 was misread as 83. Find the correct mean mark.

Solution: $\frac{\Sigma x}{100} = 40$ $\therefore \Sigma x = 4000$

Now, correct $\Sigma x = 4000 - 83 + 53 = 3970$

$\therefore$ correct mean = $\frac{\Sigma x}{100} = \frac{3970}{100} = 39.7$

18. Find A. M., G. M. and H. M. OF 3,6,24,48.

Solution: A. M. = $\frac{3+6+24+48}{4} = \frac{81}{4}$ = 20.25

G. M. = $\sqrt[4]{3 \times 6 \times 24 \times 48} = \sqrt[4]{6^4 \times 2^4}$ = 6 × 2 = 12

H. M. = $\frac{4}{\frac{1}{3}+\frac{1}{6}+\frac{1}{24}+\frac{1}{48}} = \frac{4}{\frac{16+8+2+1}{48}}$

$= \frac{4 \times 48}{27} = \frac{4 \times 16}{9}$ = 7.11

19. Find the G. M. of 4, 6, 9 with weights 1, 2, 1 respectively.

Solution: G. M. = $(4^1 \times 6^2 \times 9^1)^{\frac{1}{4}} = (2^4 \times 3^4)^{\frac{1}{4}} = 2 \times 3 = 6$

20. Find the median of 33, 86, 68, 32, 80, 48, 70, 64.

Solution: Arranging the numbers in ascending order we get

32, 33, 48, 64, 68, 70, 80, 86

Hence, n = 8 ∴ $\frac{n+1}{2} = \frac{8+1}{2} = \frac{9}{2} = 4.5$

∴ median = 4.5th term = $\frac{4th\ term + 5th\ term}{2} = \frac{64+68}{2} = 66$

21. The A. M. of 7, x -2, 10, x+3 is 9 , find the value of x .

Solution: $\frac{7+(x-2) + 10 + (x+3)}{4} = 9$ or, $\frac{2x+18}{4} = 9$

or, 2x+18 = 36 or, 2x=18 or, x=9

22. For some symmetrical distribution, Q_1 = 24 and Q_3 = 42, find the median.

Solution: For a symmetrical distribution $Q_2 - Q_1 = Q_3 - Q_2$

2 $Q_2 = Q_3 + Q_1$ or, $Q_2 = \frac{Q3 + Q1}{2} = \frac{42+24}{2}$ = 33

23. The algebraic sum of deviations of 25 observations measured from 45 is -55. What will be the A. M. of the observations?

Solution: $\sum_{i=1}^{25}(x - 45) = -55$ or, $\Sigma x - 25 \times 45 = -55$ or, $\Sigma x = 1070$

∴ A. M. = $\frac{\Sigma x}{n} = \frac{1070}{25}$ = 42.8

24. Calculate the value of x when x is the G. M. of 28 and 7

Solution: $x = \sqrt{28 \times 7}$ or, $x_2 = 28 \times 7 = (2 \times 7)^2$

$\therefore x = 2 \times 7 = 14$

25. The A. M. between two numbers is 6.5, the G. M. between them is 6, find the harmonic mean of the numbers .

Solution: A. M. × H. M. = $(\text{G. M.})^2$

$\therefore$ 6.5 × H. M. = 6^2 = 36 or, H. M. = $\frac{36}{6.5} = 5.54$

26. Find the mode of numbers:
3, 2, 5, 4, 4, 2, 4, 3, 3, 4, 4, 5, 4, 2, 4, 4, 2, 4, 5, 4, 4

Solution: Arranging the numbers in frequency distribution, we get

x:	2	3	4	5
f:	4	3	11	3

4 is the value corresponding to the highest frequency 11.

$\therefore$ Mode=4

27. Find the G. M. of the following: 1, 3, 9, 3

Solution: G. M. = $(1 \times 3 \times 9 \times 3)^{\frac{1}{4}}$ = $(3 \times 3 \times 3 \times 3)^{\frac{1}{4}}$ = 3

28. In a moderately asymmetrical distribution, the mode and mean are respectively Rs. 12.30 and Rs. 18.48. Find the median.

Solution: Mean - Mode= 3(Mean - Median)

or, 18.48 - 12.30 = 3(Mean - Median)

or, 18.48 - Median = $\frac{6.18}{3} = 2.06$

or, Median = 18.48 - 2.06 = 16.42 (Rs.)

29. If the mean of 7, (x-3), 10, (x+3) and (x-5) is 15, find x .

Solution: $\frac{7+(x-3)+10+(x+3)+(x-5)}{5} = 15$

or, $3x + 2 = 75$

or, $3x = 63$ or, $x = \frac{63}{3} = 21$

30. Find the A.M. of n numbers: 1, 3, 5 ………, (2n-1).

Solution: 1+3+5……+(2n-1)

$= \frac{n}{2}\{ 2 \times 1 + (n-1) \times 2\}$

$= \frac{n}{2} \times 2n = n^2$

∴ Mean = $\frac{Sum}{n} = \frac{n^2}{n} = n$

31. If the G. M. of x , 9, 12 be 6, find the value of x.

Solution: $(x \times 9 \times 12)^{\frac{1}{3}} = 6$

or, $x \times 9 \times 12 = 6^3$

or, $x = \frac{6\times6\times6}{9\times12} = 2$

32. Find the G. M. of 3, 6, 24, 48.

Solution: G. M. = $(3 \times 6 \times 24 \times 48)^{\frac{1}{4}}$

$= (3 \times 3 \times 2 \times 3 \times 2^3 \times 3 \times 2^4)^{\frac{1}{4}}$

$=(3^4 \times 2^8)^{\frac{1}{4}} = 3 \times 2^2 = 12$

33. If A. M. of x - 6, x - 3, x, x + 3, x + 6 is 10, find x.

Solution: $\frac{x+(x-6)+(x-3)+x+(x+3)+(x+6)}{6} = 10$

or, $\frac{6x}{6} = 10$

34. Prove that A. M. ≥ G. M. for two variables.

Solution: See Theorem 6 of chapter 2

35. Prove that G. M. ≥ H. M. for two positive numbers.

Solution: See Theorem 6 of chapter 2

36. Find the median of 94, 33, 85, 67, 32, 81, 48, 69

Solution: Arranging the numbers in ascending order,

we get 32, 33, 48, 67, 69, 81, 85, 94

By inspection we see that there are two middle terms which are 67 and 69

∴ Median = middle most value= $\frac{67+69}{2} = 68$

37. For a distribution, Arithmetic mean = Rs. 22, Mean = Rs. 20, then find the Mode.

Solution: Mean -Mode = 3(Mean - Median)

or, 22 - Mode = 3(22 - 20)

or, Mode = 22 - 6 = 16(Rs.)

38. The A.M. and G. M. of two numbers are 25 and 15 respectively, find the two numbers.

Solution: A. M. = $\frac{x_1 + x_2}{2} = 25$ or, $x_1 + x_2 = 50$

G. M. = $\sqrt{x_1 x_2} = 15$ or, $x_1 x_2 = 225$

Solving $x_1 = 5, x_2 = 45$

39. In a moderately skewed distribution, median is 30 and arithmetic mean is 27. Find the mode of the distribution.

Solution: Mean -Mode = 3(Mean - Median)

or, 27 - Mode = 3(27- 30)

or, Mode = 27 + 9 = 36

40. The A. M. of 7, p - 2, 10, p + 3 is 9; find p.

Solution: $\frac{7+(p-2)+10+(p+3)}{4} = 9$

or, $2p\ + 18\ =\ 36$ or, $2p = 18$ or, $p = 9$

41. In a moderately asymmetric distribution, the mode and mean are respectively 60.4 and 50.2. Find the median.

Solution: Median = 53.6 (using empirical relation)

42. If mean and median of a statistical distribution be respectively 14.2 and 15.4, find mode.

Solution: Mean - Mode = 3(Mean - Median)

∴ 14.2 - Mode = 3(14.2 - 15.4)

or, Mode = 14.2 + 3 × 1.2 = 17.8

43. The A. M. and G. M. of two numbers are respectively 5 and 4, find their H.M.

Solution: For two values, A. M. × H.M. = G.M.^{2}

∴ 5 × H.M. = 4^2 or, H. M. = $\frac{16}{5} = 3.2$

44. Find the median of 3, 9, 7, 4, 8, 6.

Solution: Arranging in ascending order, we get 3, 4, 6, 7, 8, 9.

Here $n = 6 \therefore \frac{n+1}{2} = \frac{6+1}{2} = \frac{7}{2} = 3.5$

Median = $\frac{n+1}{2}$th value = 3.5th value = $\frac{3rd\ value\ +v\ 4th\ value}{2} = \frac{6+7}{2} = 6.5$

45. For two positive numbers a and b (a > b) the arithmetic mean and geometric mean are 5 and 4 respectively. Find a and b.

Solution: A. M. = $\frac{a+b}{2} = 5$ and G. M. = $\sqrt{ab}$ = 4

$\therefore a + b\ =\ 10$ and $ab = 16$

$\therefore (a\ - b)^2\ =\ (a\ +\ b)^2\ -\ 4ab\ =\ 10^2\ - 4\ \times 16 = 36$

$\therefore a - b = 6$

Solving, a = 8, b = 2.

46. Find the geometric mean of 6, 9, 36, 54.

Solution: G. M. = $(6 \times 9 \times 36 \times 54)^{\frac{1}{4}}$

= $(3 \times 2 \times 3^2 \times 3^2 \times 2^2 \times 3^3 \times 2)^{\frac{1}{4}}$

= $(2^4 \times 3^8)^{\frac{1}{4}} = 2 \times 3^2 = 18$

47. For two positive values of x as x_1 and x_2 , prove that A.M. ≥ G.M.

Solution: A.M. = $\frac{x_1 + x_2}{2}$, G.M. = $\sqrt{x_1 x_2}$

Now, A.M. - G.M. = $\frac{x_1 + x_2}{2}$ - $\sqrt{x_1 x_2}$

= $\frac{x_1 + x_2 - 2\sqrt{x_1 x_2}}{2}$

= $\frac{1}{2}\{(\sqrt{x_1})^2 + (\sqrt{x_2})^2 - 2\sqrt{x_1}\sqrt{x_2}\}$

= $\frac{1}{2}(\sqrt{x_1} - \sqrt{x_2})^2 \geq 0$

∴ A.M. ≥ G.M.

48. Considering the numbers 2, 4, 8, prove that A.M. × H.M. = (G.M.)^2

Solution: A.M. = $\frac{2+4+8}{3} = \frac{14}{3}$

G.M. = $(2 \times 4 \times 8)^{\frac{1}{3}}$= $(2 \times 2^2 \times 2^3)^{\frac{1}{3}}$ = $(2^6)^{\frac{1}{3}}$ = 2^2 = 4

H.M. = $\frac{3}{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}} = \frac{3}{\frac{4+2+1}{8}} = \frac{24}{7}$

∴ A.M. × H.M= $\frac{14}{3} \times \frac{24}{7} = 16 = 4^2 = (G.M.)^2$.

49. If 6 be the geometric mean of the numbers a, 4, 8; find the value of a .

Solution: G.M. = $(a \times 4 \times 8)^{\frac{1}{3}}$= 6

or, $a \times 4 \times 8 = 6^3 = 6 \times 6 \times 6$

∴ $a = \frac{6\times6\times6}{4\times8} = \frac{27}{4}$

50. Find the mode of the n umbers: 4, 3, 2, 5, 3, 4, 5, 1, 7, 3, 2, 1.

Solution: Frequency distribution of the numbers is

x:	1	2	3	4	5	7	Total
f:	2	2	3	2	2	1	12

Mode = Value with maximum frequency = 3(maximum frequency = 3)

51. Find the median of the numbers 1,2,3……10.

Solution: Here $n = 10$ $\therefore \frac{n+1}{2} = 5.5$

Median = 5.5th value = $\frac{5th\ value + 6th\ value}{2}$

$= \frac{5+6}{2} = \frac{11}{2} = 5.5$

52. Find the Harmonic Mean of the numbers 18, 25, 30.

Solution: H.M. = $\frac{3}{\frac{1}{18}+\frac{1}{25}+\frac{1}{30}} = \frac{675}{29} = 23.28.$

53. Find the median of 33, 39, 37, 34, 38, 36.

Solution: Arranging the given numbers in ascending order we get,

33, 34, 36, 37, 38, 39

Here n = 6

$\therefore$ median = $\frac{n+1}{2} th\ value = \frac{6+1}{2} th\ value = 3.5 th\ value$

$= \frac{3rd\ value + 4th\ value}{2} = \frac{36+37}{2} = 36.5$

54. A man obtained the mean of 100 observations as 40. It was found later that he had wrongly copied 2 observations as 50 and 45. The correct figure being 40 and 48. Find the corrected mean.

Solution: $\frac{\Sigma x}{100} = 40$ or, $\Sigma x = 40 \times 100 = 4000$

∴ Correct $\Sigma x = 4000 - (50 + 45) + (40 + 48) = 3993$

∴ Correct mean = $\frac{Corrected \Sigma x}{100} = \frac{3993}{100} = 39.93$

55. If A.M. of 7, x - 2, 10, x + 3 is 9 find x.

Solution: $\frac{7+(x-2)+10+(x+3)}{4} = 9$

or, $\frac{2x+18}{4} = 9$ or, $2x+18 = 36$ or, $2x = 18$ or, $x = \frac{18}{2} = 9$.

CHAPTER-3

1. Find the variance of 2, 5, 8

Solution: $\Sigma x^2 = 2^2 + 5^2 + 8^2 = 93$ $\Sigma x = 2 + 5 + 8 = 15, n = 3$

∴ Variance = $\frac{\Sigma x^2}{n} - (\frac{\Sigma x}{n})^2 = \frac{93}{3} - (\frac{15}{3})^2 = 31 - 25 = 6$

2. Define standard deviation.

Solution: Standard deviation (S.D.) of a set of values is the positive square root of the mean of the squares of all the deviations of the values from their mean.

∴ standard deviation = $\sqrt{\frac{\Sigma(x_1 - x_2)^2}{n}}$

3. If n = 10, $\Sigma x = 120$ and $\Sigma x^2 = 1690$, find S.D.

Solution: S.D. = $\sqrt{\frac{\sum x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

= $\sqrt{\frac{1690}{10} - (\frac{120}{10})^2} = \sqrt{169 - 144} = \sqrt{25} = 5$

4. Comment on the following statement. ‘Coefficient of variation has the unit same as that of the observations.’

Solution: The statement is not correct, since co-efficient of variation has no unit.

5. What is Co-efficient of variation?

Solution: Co=efficient of variation is a relative measure of dispersion. It is expressed as percentage of mean.

Its formula is:

C.V. = $\frac{S.D.}{Mean} \times 100\%$

It has no unit.

6. Find the C.V. if mean = 168cm and S.D. = 2cm.

Solution: C.V. = $\frac{S.D.}{Mean} \times 100\% = \frac{2}{168} \times 100\% = 1.19\%$

7. Find the range of the daily wages of the ten persons.
(Rs.) 24, 18, 25, 16, 20, 28, 22, 17, 21, 27

Solution: Range = maximum wage - minimum wages = Rs. 28 - Rs. 16 = Rs. 12

8. Find the S.D. if in a distribution n=10, $\Sigma x = 40$ and $\Sigma x^2 = 250$.

Solution: S.D. = $\sqrt{\frac{\Sigma x^2}{n} - (\frac{\Sigma x}{n})^2}$= $\sqrt{\frac{250}{10} - (\frac{40}{10})^2} = \sqrt{25 - 16} = \sqrt{9} = 3$

9. If the mean and standard deviation of a distribution are Rs. 105 and Rs. 21 respectively, then find C.V.

Solution: C.V. = $\frac{S.D.}{Mean} \times 100\% = \frac{21}{105} \times 100\% = 20\%$

10. Find the mean deviation about median of the following numbers: 1, 5, 8. 3. 2.

Solution: Arranging in ascending order we get 1, 2, 3, 5, 8

∴median(M)= 3 and absolute deviation about median ($|x - M|$) = 2, 1, 0, 2, 5

∴M.D. about median = $\frac{\Sigma|x-M|}{N} = \frac{2+1+0+2+5}{5} = \frac{10}{2} = 2$

11. Find the C.V. OF 1, 5, 6.

Solution: Mean ($\bar{x}$) = $\frac{1+5+6}{4} = 4$

$$\Sigma(x-\bar{x})^2 = (1-4)^2 + (5-4)^2 + (6-4)^2 = 9 + 1 + 4 = 14$$

$$\therefore \text{S.D.} = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{14}{3}} = 2.16$$

$$\therefore \text{C.V.} = \frac{S.D.}{Mean} \times 100\% = \frac{2.16}{4} \times 100\% = 54\%$$

12. Find the mean deviation of 7, 9, 14, 24, 26 measured from their mean.

Solution: A.M = ($\bar{x}$) $= \frac{7+9+14+24+26}{5} = \frac{80}{5} = 16$

Absolute deviations from A.M. $(|x-\bar{x}|) = 9, 7, 2, 8, 10$

$$\therefore \text{mean division} = \frac{\Sigma|x-\bar{x}|}{n} = \frac{9+7+2+8+10}{5} = \frac{36}{5} = 7.2$$

13. Find the Quartile deviation of the following data:

12, 10, 17, 14, 19, 21, 27, 30, 32, 38, 34

Solution: Arranging the numbers in ascending order we get,

10, 12, 14, 17, 19, 21, 27, 30, 32, 34, 38

Here n = 11

$$Q_1 = \frac{n+1}{4} th\ i.e., \frac{11+1}{4} th = 3rd\ term = 14$$

$$Q_3 = \frac{3(n+1)}{4} th\ i.e., \frac{3(11+1)}{4} th = 9th\ term = 32$$

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2} = \frac{3-14}{2} = \frac{18}{2} = 9$$

14. Find S.D. of 4, 8, 10, 12, 16.

Solution: n= 5, $\Sigma x = 4 + 8 + 10 + 12 + 16 = 50$

$$\Sigma x^2 = 4^2 + 8^2 + 10^2 + 12^2 + 16^2 = 16 + 64 + 100 + 144 + 256 = 580$$

$$\therefore \text{S.D.} = \sqrt{\frac{\Sigma x^2}{n} - (\frac{\Sigma x}{n})^2} = \sqrt{\frac{580}{5} - (\frac{50}{5})^2} = \sqrt{116-100} = \sqrt{16} = 4$$

15. Find the Co-efficient of variation when variance = 4 and Mean = 40.

Solution: S.D. = $\sqrt{4 = 2}$; C.V. = $\frac{S.D.}{Mean} \times 100\% = \frac{2}{40} \times 100\% = 5\%$

16. Find the mean deviation of the following numbers w.r.t. mean 7, 9, 14, 24, 26.

Solution: Mean $(\overline{x}) = 16$,

M.D. = $\frac{\Sigma|x - \overline{x}|}{n}$ = $\frac{|7-16|+|9-16|+|14-16|+|24-16|+|26-16|}{5}$

= $\frac{9+7+2+8+10}{5} = \frac{36}{5} = 7.2$

17. Two variables x and y are related by y=10-3x. If the S.D. of x is 4 , what will be the S.D. of y ?

Solution: y=10-3x $\therefore \sigma_y = 3\sigma_x = 3 \times 4 = 12$

18. Find the mean if C.V. = 5% and variance = 4 .

Solution: S.D. = $\sqrt{variance} = \sqrt{4} = 2$; C.V= $\frac{S.D.}{Mean} \times 100\%$ or, 5 =$\frac{2}{mean} \times 100$

Mean = $\frac{2\times100}{5} = 40$

19. In a distribution n = 10 , $\Sigma x = 20$ and $\Sigma x^2 = 200$, then find the value of S.D.

Solution: S.D. = $\sqrt{\frac{\Sigma x^2}{n} - (\frac{\Sigma x}{n})^2}$

= $\sqrt{\frac{200}{10} - (\frac{20}{10})^2} = \sqrt{20 - 4} = \sqrt{16} = 4$

20. Find the mean deviation of the A.M. of the numbers
31, 35, 29, 63, 55, 72, 37

Solution: Mean $(\underline{x}) = \frac{31+35+29+63+55+72+37}{7} = 46$

$\Sigma|x - \underline{x}| = |31 - 46| + |35 - 46| + |29 - 46| + |63 - 46| + |55 - 46| +$

$|72 - 46| + |37 - 46|$

= 15 + 11 + 17 + 17 + 9 + 26 + 9 = 104

$\therefore$ Mean deviation = $\frac{\Sigma|x - \overline{x}|}{7} = \frac{104}{7} = 14.86(approx.)$

21. Find the mean deviation about median of the following data:
46, 79, 26, 85, 39, 59, 73

Solution: Arranging in ascending order we get 26, 39, 46, 59, 73, 79, 85

Median (M_e) = Middle most value = 4th value 59 (n=7)

$\Sigma|x - M_e|$= $|26 - 59| + |39 - 59| + |46 - 59| + |59 - 59| + |73 - 59|$

$+|79 - 59| + |85 - 59|$

= 33 + 20 + 13 + 0 + 14 + 20 + 26 = 126

∴ Mean deviation about median = $\frac{\Sigma|x-M_e|}{7} = \frac{126}{7} = 18$

22. C.V. =60% and variance = 36, find Mean.

Solution: S.D. = $\sqrt{variance} = \sqrt{36} = 6$

C. V. = $\frac{S.D.}{Mean} \times 100\%$

or, 60 =$\frac{6}{mean} \times 100$ or, Mean = 10

23. For a distribution A.M. is 40 and variance is 100. Find C.V.
S.D. = $\sqrt{variance} = \sqrt{100} = 10$

Solution: C.V.= $\frac{S.D.}{Mean} \times 100\%$ = $\frac{10}{40} \times 100\%$ = 25%

24. Find the standard deviation of 2, 4, 5, 6, 8.

Solution: Here n = 5

$\Sigma x = 2 + 4 + 5 + 6 + 8 = 25$ and $\Sigma x^2 = 2^2 + 4^2 + 5^2 + 6^2 + 8^2 = 145$

∴ S.D. = $\sqrt{\frac{\Sigma x^2}{n} - (\frac{\Sigma x}{n})^2}$= $\sqrt{\frac{145}{5} - (\frac{25}{5})^2} = \sqrt{29 - 25} = \sqrt{4} = 2.$

25. If y = 12 - 5x and is standard deviation of x is 7, then find the standard deviation of y .

Solution: If y = a + bx, then $(S.D.)_y$ = $|b|(S.D.)_x$.

$\therefore$ (S.D.)$_y$ = $|-5|(S.D.)_x = 5 \times 7 = 35$

26. Find the standard deviation of 4, 8, 10, 12 and 16.

Solution: Here n = 5, Mean $(\underline{x}) = \frac{4+6+8+10+12+16}{5} = 10$

$$\therefore \text{Variance} = \frac{(4-10)^2 + (8-10)^2 + (10-10)^2 + (12-10)^2 + (16-10)^2}{5}$$

$$= \frac{36+4+0+4+36}{5} = \frac{80}{5} = 16$$

$\therefore$ S.D. = $\sqrt{variance} = \sqrt{16} = 4$

27. Find the standard deviation of the numbers 49, 63, 46, 59, 65, 52, 60, 54.

Solution: Arranging the numbers in ascending order, we get 46, 49, 52, 54, 59, 60, 63, 65.

Here, n = 8 and $(\underline{x}) = \frac{46+49+52+54+59+60+63+65}{8} = \frac{448}{8} = 56$

$$\text{Variance} = \frac{(46-56)^2+(49-56)^2+(52-56)^2+(54-156)^2+(59-56)^2+(60-56)^2+(63-56)^2+(65-56)^2}{8}$$

$$= \frac{10^2+7^2+4^2+2^2+3^2+4^2+7^2+9^2}{8}$$

$$= \frac{100+49+16+4+9+16+49+81}{8} = \frac{324}{8}$$

$\therefore$ S.D. = $\sqrt{variance} = \sqrt{\frac{324}{8}} = \frac{9\sqrt{2}}{2} = 6.36.$

28. Two variables x and y are related by y = 14 - 3x, If $\sigma_y = 9$, find σ_x.

Solution: $\sigma_y = |-3| \times \sigma_x = 3 \times \sigma_x$

$\therefore 9 = 3 \times \sigma_x$ or, $\sigma_x = \frac{9}{3} = 3.$

29. Find the mean if coefficient of variation = 15% and variance = 4.

Solution: C.V.= $\frac{S.D.}{Mean} \times 100\%$

Now, S.D. = $\sqrt{variance} = \sqrt{4} = 2$

$\therefore 15 = \frac{2}{mean} \times 100$

or, Mean = $\frac{200}{15} = 13.33$

30. Find the quartile deviation of 22, 17, 25, 20, 29, 27 and 35.

Solution: Arranging in ascending order we get 17, 20, 22, 25, 27, 29, 35.

Here n = 7 $\therefore \frac{n+1}{4} = \frac{7+1}{4} = \frac{8}{4} = 2, \frac{2(n+1)}{4} = \frac{3(7+1)}{4} = \frac{3\times8}{4} = 6$

$\therefore Q_1 = 2nd\ term = 20, Q_3 = 6th\ term = 29$

$\therefore$ Quartile deviation = $\frac{Q_3 - Q_1}{2} = \frac{29-20}{2} = \frac{9}{2} = 4.5$

31. Compute standard deviation from the following observations:
61, 52, 37, 89, 28, 93, 65

Solution: Calculation for S.D.

x	y = x - A	y^2
28	-36	1296
37	-27	729
52	-12	144
61	-3	9
64 = A	0	0
65	1	1
89	25	625
93	29	841
Total	$\Sigma y = 55 - 78$ = -23	Σy^2 = 3645

$\therefore$ S.D. = $\sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}$

= $\sqrt{\frac{3645}{8} - (\frac{-23}{8})^2}$

= $\sqrt{455.625 \; - \; (2.875)^2}$

= $\sqrt{455.625 - 8.265}$

= $\sqrt{447.36} \quad = 21.15$

CHAPTER - 4

1. If the second and third moment of a distribution are respectively 16 and -12.8, find the moment measure of co-efficient of skewness.

Solution: Here, $m_2 = \; 16 \, , m_3 = -12.8,$

Co-efficient of skewness = $\frac{m_3}{\sqrt{{m_2}^3}} = \frac{-12.8}{\sqrt{16^3}} = \frac{-12.8}{64} = \; -0.2$

2. If the first two moments of a distribution about 4 are μ'_1 and μ'_2; if μ'_1= 1.5, find mean ($\underline{x}$). Again if μ'_2= 17, find μ_2.(2nd moment about mean).

Solution: ($\underline{x}$) = 4 + μ'_1= 4 - 1.5 = 2.5

μ_2= μ'_2 - μ'_1= 17-$(-1.5)^2$ = 17 - 2.25 = 14.75

3. If Q_1= 25, Q_2=36 and Q_3 = 45, find the measure of skewness.

Solution: Skewness = $\frac{Q_3 \, - \, 2Q_2 \, + Q_1}{Q_3 - Q_1} \; = \; \frac{45 - 2 \times 36 + 25}{45 - 25} = \frac{-2}{20} = \; -0.1$

4. For a symmetrical distribution (where skewness = 0) Q_1= 24 and Q_2 = 42. Using formula to Bowley's measure of skewness, find the median of two distributions.

Solution: According to Bowley's formula skewness = $\frac{Q_3 \, - \, 2Q_2 \, + Q_1}{Q_3 - Q_1}$

or, 0 = $\frac{42 \, - \, 2Q_2 \, + 24}{42 \, - \, 24}$

or, $0 = \frac{66 - 2Q_2}{18}$

or, $66 - 2Q_2 = 0$ or, $Q_2 = 33$

∴ median = 33.

5. For a moderately skewed distribution, mean = 172, median = 167 and S.D. = 60; find the coefficient of skewness.

Solution: Co-efficient of skewness = $\frac{3(mean-median)}{S.D.}$

$= \frac{3(172-167)}{60} = \frac{3\times5}{60} = 0.25$

6. If the 2nd and 3rd central moments of a distribution be 4 and 12 respectively, find the skewness of the distribution.

Solution: Here, $m_2 = 4, m_3 = 12,$

Now, skewness = $\frac{m_3}{\sqrt{{m_2}^3}} = \frac{12}{\sqrt{4^3}} = \frac{12}{8} = 1.5$

7. If the first moment of a distribution about the value 2 is 2; find the Mean.

Solution: mean = A + $m'_1 = 2 + 2 = 4.$

8. If the second and third central moments of distribution be 4 and 10, find the skewness of the distribution.

Solution: Here, $m_2 = 4, m_3 = 10,$

∴ skewness = $\frac{m_3}{\sqrt{{m_2}^3}} = \frac{10}{\sqrt{4^3}} = \frac{10}{8} = 1.25$

9. For some symmetrical distribution (where skewness = 0) Q_1= 36 and Q_2 = 63. Using formula to Bowley's measure of skewness, find the median of two distributions.

Solution: $0 = \frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1}$

or, $Q_3 = 2Q_2 + Q_1 = 0$

or, $Q_2 = \frac{Q_3+Q_1}{2} = \frac{63+36}{2} = 49.5$

∴ median = 49.5

10. Give the expression for the measurement of Kurtosis.

Solution: Kurtosis = $\frac{m_4}{{m_2}^2} - 3$ where m_2 and m_4 are 2nd and 4th central moments respectively.

11. Find the Pearson's measure of skewness when Mean = 86, Median = 80 and S.D. = 43.

Solution: Co-efficient of skewness = $\frac{3(mean - median)}{S.D.} = \frac{3(86-80)}{43} = \frac{3\times6}{43} = 0.42$

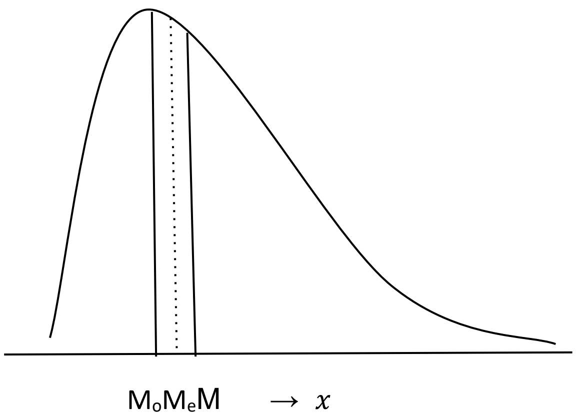
12. For a positively skewed distribution which one is true:

(i) Mean > Median > Mode

(ii)Mean < Median < Mode

Put the correct statement in the form of a diagram.

Solution: (i) is true.



13. If Q_1 = 26, Q_3 = 76 and coefficient of skewness 0.2, find the median.

Solution: skewness = $\frac{Q_1 + Q_3 - 2\,Q_2}{Q_3 - Q_1}$ ∴ $0.2 = \frac{26+76-2\,Q_2}{76-26}$

or, $0.2 \times 50 = 102 - 2\,Q_2$ or, 2 Q_2=102 -10=92 or, $Q_2 = \frac{92}{2} = 46$ ∴ median (Q_2) = 46.

14. What do you mean by Leptokurtic and Meso-Kurtic frequency curve?

Solution: If a frequency curve is highly peaked, then it is called Leptokurtic and if a frequency curve is highly flat, then it is called Meso-Kurtic.

15. Find the first two moments about zero for the set of numbers 1, 3, 5, 7.

Solution: $m_1 = \frac{\Sigma x}{n} = \frac{1+3+5+7}{4} = 4$

$m_2 = \frac{\Sigma x^2}{n} = \frac{1^2+3^2+5^2+7^2}{4} = \frac{84}{4} = 21$

16. For a moderately skewed distribution, Mean = 172, Median = 167 and S.D.=60. Find the coefficient of skewness.

Solution: skewness = $\frac{3(mean - median)}{S.D.} = \frac{3(172 - 167)}{60} = \frac{3 \times 5}{60} = \frac{1}{4} = 0.25$

17. The Karl-Pearson's Coefficient of skewness of a distribution is 0.32. Its standard deviation is 6.5 and mean 29.6. Find the mode.

Solution: Skewness = $\frac{Mean - Mode}{S.D.}$

∴ 0.32 = $\frac{29.6 - Mode}{6.5}$

or, 29.6 - Mode = 0.32 × 6.5 = 2.08

or, Mode = 29.6 -2.08 = 27.52

18. In a group of 10 observations $\Sigma x = 452, \Sigma x^2 = 24270$ and $mode = 43.7$. Find the Pearson's Coefficient of skewness.

Solution: Mean = $\frac{\Sigma x}{n} = \frac{452}{10} = 45.2$

S.D. = $\sqrt{\frac{\Sigma x^2}{n} - (\frac{\Sigma x}{n})^2}$

= $\sqrt{\frac{24270}{10} - (\frac{452}{10})^2} = \sqrt{2427 - (45.2)^2} = \sqrt{2427 - 2043.04} = \sqrt{383.96} = 19.6$

$\therefore$skewness = $\frac{Mean - Mode}{S.D.}$

= $\frac{45.2 - 43.7}{19.6} = \frac{1.5}{19.6} = 0.0765.$

19. For moderately skewed distribution mean = 172, median = 167 and S.D. = 60. Find the coefficient of skewness and mode of the distribution.

Solution: Mean - Mode = 3(Mean - Median)

or, 172 - Mode = 3(172-167) = 2 * 5 = 15

or, Mode = 172 - 15 = 157

$\therefore$skewness = $\frac{Mean - Mode}{S.D.}$

= $\frac{172-157}{60} = \frac{15}{60} = 0.25.$

20. The first two moments of a distribution about 1 are 2 and 25. Find mean and S.D. of the distribution.

Solution: $m_1' = 2, m_2' = 25, A = 1$

Mean $(\bar{x}) = A + m_1' = 1 + 2 = 3$

S.D. (σ) = $\sqrt{m_2' - m_1'^{\,2}} = \sqrt{25 - 2^2} = \sqrt{21} = 4.58$

21. If the 2nd and 4th central moments of a distribution be 5 and 75 respectively, find the kurtosis of the distribution.

Solution: m_2=5, m_4=75

Now, kurtosis = $\frac{m_4}{m_2^{\,2}} - 3 = \frac{75}{5^2} - 3 = \frac{75}{25} - 3 = 3 - 3 = 0.$

22. In a distribution the difference between first and third quartiles is 2.03 and their sum is 72.67; find the skewness if the median is 36.18.

Solution: $Q_3 - Q_1 = 2.03, Q_3 + Q_1 = 72.67, and\ Q_2 = 36.18$

Skewness = $\frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1} = \frac{72.67 - 2\times36.18}{2.03} = \frac{72.67-72.36}{2.03} = \frac{0.31}{2.03} = 0.153.$

23. The first three moments of a distribution about 3 are respectively 4, 65, 134. Find the arithmetic mean, standard deviation and moment measure of skewness of the distribution.

Solution: A= 3, $m_1{}'$ = 4, $m_2{}'$ = 65, $m_3{}'$ = 134

$\therefore$ Mean $(\bar{x}) = A + m_1{}' = 3 + 4 = 7$

$m_2 = m_2{}' - (m_1{}')^2 = 65 - 4^2 = 65 - 16 = 49$

$m_3{}' = m_3{}' - 3\, m_2{}'\, m_1{}' + 2(\, m_1{}')^3$

$= 134 - 3 \times 65 \times 4 + 2 \times 4^3$

=134 - 780 + 128 = -518

$\therefore$S.D. $(\sigma) = \sqrt{m_2} = \sqrt{49} = 7,$

Skewness = $\frac{m_3}{(m_2)^{\frac{3}{2}}} = \frac{-518}{(49)^{\frac{3}{2}}} = \frac{-518}{7^3} = \frac{-518}{343} = -1.51$

24. If the first moment of a distribution about the value 2 is 4, find the mean of the distribution.

Solution: Here, $A = 2,\ m_1' = 4$

$\therefore Mean(\bar{x}) = A + m_1' = 2 + 4 = 6$

25. Write the formula of measure of skewness according to Bowley.

Solution: According to Bowley,

Co-efficient of skewness = $\frac{Q_3+Q_1-2Q_2}{Q_3-Q_1}$

Where Q_1, Q_2 and Q_3 are the 1st, 2nd, and 3rd quantiles respectively.