Normal Distribution

A continuous random variable x is said to have a normal distribution with parameters μ(-∞<μ<∞) and σ(>0) if its probability distribution function is given by

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad , -\infty< x <\infty.$$

In this case, we say x is a normal variate with parameters μ and σ and is denoted by $x \sim N\ (\mu, \sigma)$.

Observe that

i. $f(x) \geq 0\ \forall\ x$

ii. $\int_{-\infty}^{\infty} f(x)\ dx\ = \int_{-\infty}^{\mu} f(x)\ dx + \int_{\mu}^{\infty} f(x)\ dx$

$$= \lim_{B_1\to\infty} \int_{-B_1}^{\mu} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\ dx$$

$$+ \lim_{B_2\to\infty} \int_{\mu}^{B_2} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\ dx$$

$$= 1$$

So, this is a valid probability distribution.

Distribution Function

$$F(x) = P(x \leq x) = \int_{-\infty}^{x} f(t)\ dt = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-(t-\mu)^2}{2\sigma^2}}\ dt$$

Theorem 1: If x is a normal variate with parameters μ and σ then

i. Mean = E(x) = μ

ii. Var(x) = σ^2

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad , -\infty< x <\infty.$$

Mean = E(x) = $\int_{-\infty}^{\infty} xf(x)\ dx$

$$= \int_{-\infty}^{\infty}(x-\mu)f(x)\ dx + \ \mu\int_{-\infty}^{\infty} f(x)\ dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(x-\mu)e^{(x-\mu)^2/2d^2}\ dx + \mu \qquad [\int_{-\infty}^{\infty} f(x)\ dx = 1]$$

Now, $\int_{-\infty}^{\infty}(x-\mu)e^{-\frac{(x-\mu)^2}{2\sigma^2}}\ dx$

$$= \lim_{\substack{B_1\to\infty \\ B_2\to\infty}} \int_{-B_1}^{B_2}(x-\mu)e^{-\frac{(x-\mu)^2}{2\sigma^2}}\ dx \qquad [\quad \text{Let} \quad z = (x-\mu)^2 \quad]$$

$$= \lim_{\substack{B_1\to\infty \\ B_1\to\infty}} \frac{1}{2}\int_{(B_1+\mu)^2}^{(B_2-\mu)^2} \frac{e^{-z}}{2\sigma^2}\ dz \qquad [\quad => (x-\mu)dx = \frac{1}{2}\ dz \quad]$$

$$= \lim_{\substack{B_1\to\infty \\ B_2\to\infty}} \frac{1}{2}\left[-2\sigma^2 e^{-\left(\frac{z}{2\sigma^2}\right)}\right]_{(B_1+\mu)^2}^{(B_2-\mu)^2}$$

$$= \lim_{\substack{B_1\to\infty \\ B_2\to\infty}} \sigma^2\left[e^{-\frac{(B_1+\mu)^2}{2\sigma^2}} - e^{-\frac{(B_2-\mu)^2}{2\sigma^2}}\right] = 0$$

∴ Mean = E(X) = μ

$$\text{Var(X)} = E\{(X-\mu)^2\}$$

$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(x-\mu)^2 e^{-(x-\mu)^2/2\sigma^2}\,dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\mu}(x-\mu)^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}}\frac{dx}{\sigma\sqrt{2}} + \frac{1}{\sqrt{\pi}}\lim_{B_1\to\infty}\int_{-B_1}^{\mu}(x-\mu)^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}}\frac{dx}{\sigma\sqrt{2}}$$

$$= \frac{1}{\sqrt{\pi}}\lim_{B_1\to\infty}\int_{\frac{B_1+\mu}{\sigma\sqrt{2}}}^{0} -2\sigma^2 u^2 e^{-u^2}du + \frac{1}{\sqrt{\pi}}\lim_{B_2\to\infty}\int_0^{\frac{B_2-\mu}{\sigma\sqrt{2}}} 2\sigma^2 v^2 e^{-v^2}\,dv$$

$$\left[u = -\left(\frac{x-\mu}{\sigma\sqrt{2}}\right),\ v = \frac{x-\mu}{\sigma\sqrt{2}}\right]$$

$$= \frac{2\sigma^2}{\sqrt{\pi}}\lim_{B_1\to\infty}\int_0^{\frac{B_1+\mu}{\sigma\sqrt{2}}} u^2 e^{u^2}du + \frac{2\sigma^2}{\sqrt{\pi}}\lim_{B\to\infty}\int_0^B 2u^2 e^{-u^2}\,du$$

$$= \frac{4\sigma^2}{\sqrt{\pi}}\int_0^{\infty} u^2 e^{-u^2}\,du + \frac{2\sigma^2}{\sqrt{\pi}}\lim_{B_2\to\infty}\int_0^{\frac{B_2-\mu}{\sigma\sqrt{2}}} v^2 e^{-v^2}\,dv$$

$$= \frac{2\sigma^2}{\sqrt{\pi}}\int_0^{\infty} u^2 e^{-u^2}\,du = \frac{2\sigma^2}{\sqrt{\pi}}\lim_{B\to\infty}\int_0^B 2u^2 e^{-u^2}du$$

$$= \frac{2\sigma^2}{\sqrt{\pi}}\lim_{B\to\infty}\int_0^{B^2} e^{-z}z^{\frac{1}{2}}\,dz \qquad [u^2 = z \quad => 2udy = dz \quad => u = z^{\frac{1}{2}}]$$

$$= \frac{2\sigma^2}{\sqrt{\pi}}\int_0^{\infty} e^{-z}z^{\frac{3}{2}-1}\,dz = \frac{2\sigma^2}{\sqrt{\pi}}\Gamma\left(\frac{3}{2}\right) = \frac{2\sigma^2}{\sqrt{\pi}}\Gamma\left(\frac{1}{2}\right) \qquad [\because \Gamma(n+1) = n\Gamma(n)]$$

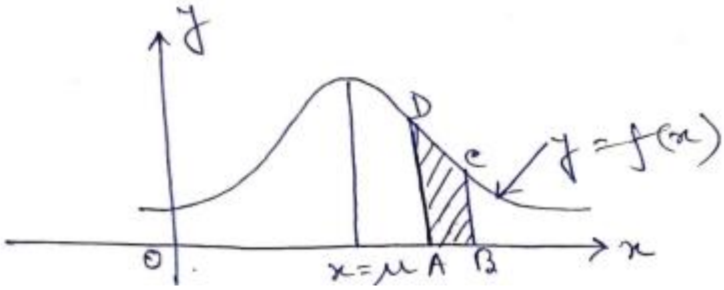
$$= \frac{\sigma^2}{\sqrt{\pi}} \times \sqrt{\pi} = \sigma^2$$

∴ S.D of X = σ

Normal Probability Curve or Normal Curve

The graph of the probability density function of a normal variate is called normal probability curve or normal curve. So, the normal probability curve with mean μ and variance σ^2 is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-u)^2}{2\sigma^2}}, -\infty < x < \infty$$



i) The curve is bell shaped and symmetrical about the highest ordinate, i.e., about the line $x = \mu$.
ii) The maximum probability occurs around the point $x = \mu$.
iii) Since $f(x) \geq 0$, it can never be negative and hence no portion of the curve lies below x-axis.
iv) x-axis is an asymptote to this curve.
v) Area property: If A and B are any 2 points on the x-axis, the shaded area, ABCD, bounded by the curve, the ordinate at A and B and the x-axis is equal to the probability that x lies between A and B.

Theorem 2: If a continuous random variable X has normal distribution with parameters μ and σ, then the random variable $Z = \frac{X-\mu}{\sigma}$ has standard normal distribution.

X~N (μ, σ) and $Z = \frac{X-\mu}{\sigma}$ $=> X = \mu + \sigma Z$

Now, $F(z) = P(Z \leq z) = P\left(\frac{X-\mu}{\sigma} \leq z\right) = P(X \leq \mu + \sigma Z)$

$$\therefore F(z) = \int_{-\infty}^{u+\sigma z} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \lim_{B\to\infty} \int_{-B}^{\mu+\sigma z} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sqrt{2\pi}} \lim_{B\to\infty} \int_{-\left(\frac{B+\mu}{\sigma}\right)}^{z-B} e^{-\left(\frac{t^2}{2}\right)} dt \qquad [t = \frac{x-\mu}{\sigma}]$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\left(\frac{t^2}{2}\right)} dt$$

$$f(z) = \frac{dF}{dz} = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{z^2}{2}\right)}, \quad -\infty < z < \infty$$

Hence, z~ N (0,1).

1. X is a normal variate with mean 30 and variance 25. Find
   i. P ($26 \leqslant X \leqslant 40$),
   ii. P ($X<35$)
   iii. P ($X \geqslant 45$), and
   iv. P ($|x-30| > 5$)

$X \sim N(\mu, \sigma), \mu = 30, \quad \sigma^2 = 25 \quad => \sigma = 5$

$$Z = \frac{X-\mu}{\sigma} = \frac{X-30}{5} \ \therefore Z \sim N(0,1)$$

i. X=26 $=> Z = \frac{26-30}{5} = -0.8$

X=40 $=> Z = \frac{40-30}{5} = 2$

$\therefore P(26 \le X \le 40) = P(-0.8 \le Z \le 2)$

$= P(-0.8 \le Z \le 0) + P(0 < Z \le 2)$

$= P(0 \le Z \le 0.8) + P(0 < Z \le 2)$

$= 0.2881 + 0.4772 \quad = 0.7653$

ii. P(X<35) = P(Z<1)

$= P(\infty < Z \le 0) + P(0 < Z < 1)$

$= 0.5 + 0.3413$

$= 0.8413$

iii. $P(X \ge 45) = P(Z \ge 3)$

$= P(0 \le Z < \infty) - P(0 \le Z \le 3)$

$= 0.5 - 0.49865$

$= 0.00135$

iv. P ($|x-30| > 5$)

$= 1 - P(|x-30| \le 5)$

$= 1 - P(-5 \le x - 30 \le 5)$

$= 1 - P(25 \le x \le 35)$

$= 1 - P(-1 \le Z \le 1)$

$= 1 - 2P(0 \le Z \le 1)$

$= 1 - 2 \times 0.3413 = 1 - 0.6826 = 0.3174$

Normal Approximation to Binomial Distribution

Theorem: Let the random variable X follows binomial distribution with probability mass function

$$P(X = i) = C_i^n p^i (1-p)^{n-i}, i = 0, 1, 2, \dots, n$$

where n and p are parameters.

If $n \to \infty$ and p is not very small but of moderate magnitude, then the distribution of the random variable

i. X approaches to $N\left(np, \sqrt{np(1-p)}\right)$.
ii. $Z = \frac{x-np}{\sqrt{np(1-p)}}$ approaches to N (0, 1).

1. If fair coin is tossed 400 times. Using normal approximation to binomial distribution find the probability of obtaining i) exactly 200 heads, ii) between 190 and 210 heads, both inclusive. Given that the area under standard normal curve between z=0 and z=0.05 is 0.0199 and between z=0 and z=1.05 is 0.3531.

   Since n is large, we suppose that X is an approximately normal variate with parameters:

   $$\mu = np = 400 \times \frac{1}{2} = 200$$

   $$\sigma = \sqrt{np(1-p)} = \sqrt{400 \times \frac{1}{2} \times \frac{1}{2}} = 10$$

   So, $Z = \frac{x-\mu}{\sigma} = \frac{X-200}{10}$ is approximately standard normal variate.

   i) When $X = 199.5$, $Z = \frac{199.5-200}{10} = -0.05$

   When X = 200.5, Z=0.05

   $\therefore P(X = 200) = P(199.5 \le X \le 200.5)$

   $=P(-0.05 \le Z \le 0.05)$

   =2×0.199 =0.0398

   ii) When X=189.5, $Z = \frac{189.5-200}{10} = -1.05$

   When X=210.5, Z=1.05

   $P(190 \le X \le 210) = P(189.5 \le X \le 210.5)$

   $=P(-1.05 \le Z \le 1.05)$

   $=2 \times P(0 \le Z \le 1.05)$

   $= 2 \times 0.3531$ =0.7062

1. If X follows a normal distribution with mean 12 and variance 16, find $P(X \geq 20)$

[Given: $\int_{-\infty}^{2} \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{t^2}{2}\right)} dt = 0.977725$]

X=20, $Z = \frac{20-12}{4} = 2$

$\therefore P(X \geq 20) = P(Z \geq 2)$

$= 1 - P(Z < 2)$

$= 1 - \int_{-\infty}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$

$= 1 - 0.977725 \quad = 0.022275$

2. If X is normally distributed with zero mean and unit variance find the expectation of x^2.

E(X)=0, Var(X) = 1

⇨ $E(X^2) - \{E(X)\}^2 = 1$

⇨ $E(X^2) = 1$

3. If X is normally distributed with mean 3 and s.d 2, find c such that $P(X > C) = 2P(X \leq C)$. Given that,

$$\int_{-\infty}^{0.43} \emptyset(t) dt = 0.6666$$

$Z = \frac{X-\mu}{\sigma} = \frac{X-3}{2}$ X=C $=> Z = \frac{C-3}{2}$

$\therefore P(X > C) = P\left(Z > \frac{C-3}{2}\right)$

$= 1 - P\left(Z \leq \frac{C-3}{2}\right)$

$= 1 - \int_{-\infty}^{\frac{C-3}{2}} \emptyset(t)\, dt$

$P(X \leq C) = P\left(Z \leq \frac{C-3}{2}\right)$

$= \int_{-\infty}^{\frac{C-3}{2}} \emptyset(t) dt$

Given: $P(X > C) = 2P(X \leq C)$

⇨ $1 - \int_{-\infty}^{\frac{C-3}{2}} \emptyset(t) = 2 \times \int_{-\infty}^{\frac{C-3}{2}} \emptyset(t)\, dt$

⇨ $\int_{-\infty}^{\frac{C-3}{2}} \emptyset(t)\, dt = \frac{1}{3}$ ------ (1)

Given: $P(Z \leq 0.43) = \int_{-\infty}^{0.43} \emptyset(t)\, dt = 0.6666$

$\therefore P(-0.43 \leq Z \leq 0 = P(0 \leq Z \leq 0.43$

$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

$\therefore P(Z \leq -0.43) = \int_{-\infty}^{-0.43} \emptyset(t)\, dt = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$ ------(2)

Comparing (1) & (2), we get,

$\frac{C-3}{2} = -0.43$

⇨ C = 3 – 0.86 = 2.14 (Ans)

4. If a random variable X follows normal distribution such that
$P(9.6 \leq X \leq 13.8) = 0.7008$ and
$P(X \geq 9.6 = 0.8159,$ where
$\int_{-\infty}^{0.9} \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{t^2}{2}\right)} dt = 0.8849$
Find the mean and variance of X.

$X \sim N(\mu, \sigma), \quad Z = \frac{X - \mu}{\sigma} \sim N(0,1)$
$P(Z \leq 0.9) = 0.8159$
$P(Z \leq -0.9) = P(Z \geq 0.9)$
$= 1 - P(Z \leq 0.9)$
$= 1 - 0.8159$
$= 0.1841$
$\therefore P(-0.9 \leq Z \leq 1.2) = P(Z \leq 1.2) - P(Z \leq -0.9)$
$= 0.8849 - 0.1841 = 0.7008$
$\Rightarrow P\left(-0.9 \leq \frac{X-\mu}{\sigma} \leq 1.2\right) = 0.7008$ ---- (1)

Given: $P(9.6 \leq X \leq 13.8) = 0.8008$

$\Rightarrow P\left(\frac{9.6-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{13.8-\mu}{\sigma}\right) = 0.7008$ ------ (2)

Comparing (1) & (2), we get,

$\frac{9.6 - \mu}{\sigma} = -0.9, \quad \frac{13.8 - \mu}{\sigma} = 1.2$

$\Rightarrow \sigma = 2, \quad \mu = 11.4$

$\therefore$ Mean = 11.4, $\quad Var(X) = 2^2 = 4$

5. The length of bolds produced by a machine is normally distributed with mean 4 and S.D 0.5. A bolt is defective if its length does not lie in the interval (3.8, 4.3). find the percentage of defective boults produced by the machine.

[$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.6} e^{-\left(\frac{t^2}{2}\right)} dt = 0.7257,$
$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{-\left(\frac{t^2}{2}\right)} dt = 0.6554$]

$X \sim N(\mu, \sigma), \quad \mu = 4, \quad \sigma = 0.5$
$Z \sim N(0,1), \quad Z = \frac{X - \mu}{\sigma}$
When X=3.8, $\quad Z = \frac{3.8-4}{0.5} = -0.4$
When X=4.3, $\quad Z = \frac{4.3-4}{0.5} = 0.6$
$\therefore P(3.8 < X < 4.3) = P(-0.4 < Z < 0.6) = P(Z \leq 0.6) - P(Z \leq -0.4)$
$P(Z \leq -0.4) = P(Z > 0.4)$
$= 1 - P(Z \leq 0.4)$
$= 1 - 0.6554 = 0.3446$

$\therefore P(3.8 < X < 4.3) = 0.7257 - 0.3446 \ = 0.3811$
∴ Percentage of non-defective bolt = (100 – 38) %
= 62% (nearly)

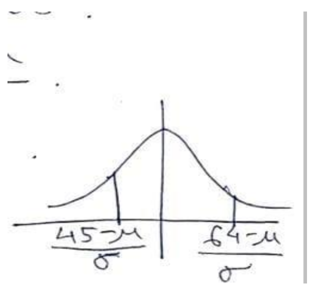
6. The marks obtained by 1000 students to a final examination are found to be approximately normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be between 60 and 75 both inclusive. Given that area under the normal curve
$\Phi(\mathrm{z}) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-\mathrm{z}^2}{2}\right)$ between z=0 and z=2 is 0.4772 and between z-=0 and z=1 is 0.3413.

When x=60, $\mathrm{z} = \frac{60-70}{5} = -2$
When x=75, $\mathrm{z} = \frac{75-70}{5} = 1$
$\therefore \mathrm{P}(60 \le \mathrm{x} \le 75) = \mathrm{P}(-2 \le \mathrm{x} \le 1)$
$= \mathrm{P}(-2 \le \mathrm{x} \le 0) + \mathrm{P}(0 \le \mathrm{x} \le 1)$
$= \mathrm{P}(0 \le \mathrm{x} \le 2) + \mathrm{P}(0 < +\mathrm{x} < +1)$
$= 0.4772 + 0.3413$
$= 0.8185$
∴ Number of required students $= 0.8185 \times 1000$
$= 818$ (nearly)

7. In a normal distribution 31% of the items are under 45 and 8% are above 64. Find the mean and standard deviation. [Given: $\mathrm{P}(0 < \mathrm{z} < 1.405) = 0.42$, $\mathrm{P}(-0.496 < \mathrm{z} < 0) = 0.19$]

$P(x < 45) = \frac{31}{100} = 0.31$

$P(x > 64) = \frac{8}{100} = 0.08$
When x=45, $z = 45 - \mu\sigma$
When x=64, $z = \frac{64-\mu}{\sigma}$
$\therefore P\left(z < \frac{45-\mu}{\sigma}\right) = 0.31$



$\frac{45-\mu}{\sigma}$ *is on negetive side as* $0.31 < 0.5$.

$\therefore P\left(\frac{45-\mu}{\sigma} < z < 0\right) = 0.5 - 0.31$
$= 0.19$ ------- (1)
Given, $P(-0.496 < z < 0) = 0.19$ -------(2)
Comparing 1 and 2, we get
$\frac{64-\mu}{\sigma} = -0.496$ ------- (3)
$P\left(\mathrm{z} > \frac{64-\mu}{\sigma}\right) = 0.08$
$\frac{64-\mu}{\sigma}$ *is on positive side as* $0.08 < 0.5$
$\therefore P\left(0 < z < \frac{64-\mu}{\sigma}\right) = 0.5 - 0.08$

$= 0.42$ -------4

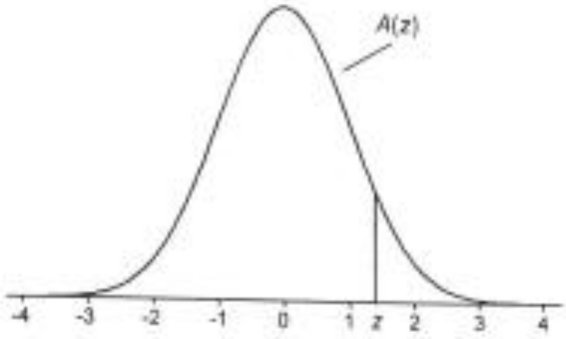
Given, $P(0 < z < 1.405) = 0.42$ -------5

Comparing 4 and 5, we get,

$\frac{64-\mu}{\sigma} = 1.405$ -------6

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							