B.COM

Business mathematics and statistics

<u>Semester III</u>

SETS : A set is a well-defined collection of distinct objects, considered as an object in its own right. The arrangement of the objects in the set does not matter. For example, the numbers 2, 4, and 6 are distinct objects when considered separately; when considered collectively, they form a single set of size three, written as {2, 4, 6}, which could also be written as {2, 6, 4}, {4, 2, 6}, {4, 6, 2}, {6, 2, 4} or {6, 4, 2}. Sets can also be denoted using capital letters such as A, B, C.

TYPES OF SETS

1) Singleton sets: A set which contains only one element is known as **Singleton set** .

Examples :

1) If P = { x | x is a prime number 10 and 12 } then P = {11}

As we observe that there is only one element in set P.
n(P) = 1

so **set P is a singleton set.**

2) If A = { x| x ∉ 3 < x < 5 } then

A = { x| x ∉ 3 < x < 5 }

A = { 4}

As the set A contains only one element so set A is a singleton set.

2) Finite sets : The sets in which number of elements are limited and can be counted, such sets are called **finite sets.**

Example :

If A= { x | x is a prime number, x<10 } then A= { 2,3,5,7}

Here then there are only 4 elements which satisfies the given condition.

Thus, **set A is a finite set.**

3) Infinite sets : The sets in which number of elements are unlimited and cannot be counted, such sets are called **infinite sets** .

Example :

set C = { 10,20,30,40,50,60,...}

As the number of elements in set C are infinity (uncountable).
Thus, **set C is an infinite set.**

4) Empty set : A set which has no element in it and is denoted by ϕ
(Greek letter 'phi')

Thus n(ϕ) = 0

It is also known as **null set** or **void set .**

Example :

set A ={ 18 < x < 19}

So between 18 and 19 there is no element.

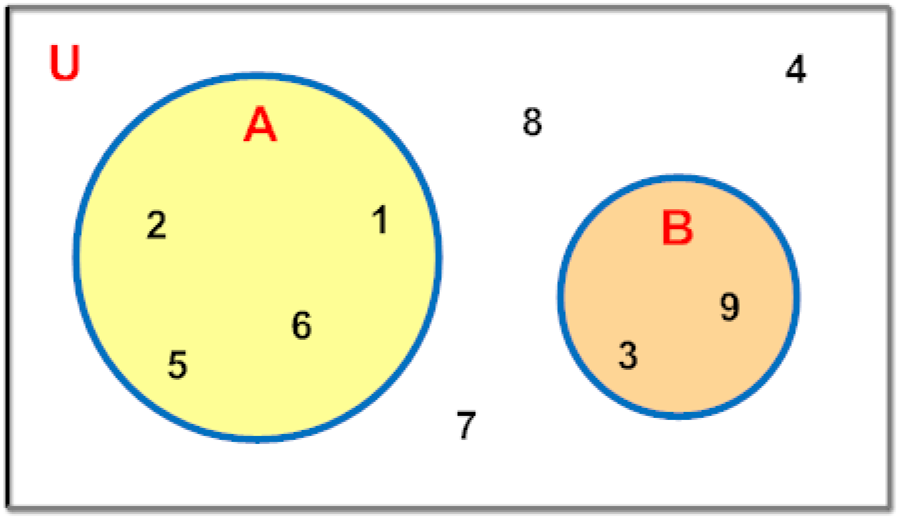
Thus, **set A is an empty set.**

SUBSET:
A is a subset of B when every member of A is a member of B.

Example: B = {1,2,3,4,5}
Then A = {1,2,3} is a subset of B
Other subsets of B include {2,3} or {1,4,5} or {4} etc...
But {1,2,6} is NOT a subset of B as it has 6 (which is not in B)

Definition: A **Universal Set** is the set of all elements under consideration, denoted by capital U.
All other sets are subsets of the universal set.

Example 2: Given U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, *A* = {1, 2, 5, 6} and *B* = {3, 9}, draw a Venn diagram to represent these sets.



OPERATION ON SETS

Union of Sets

Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12}. Then, A U B is represented as the set containing all the elements that belong to both the sets individually. Mathematically,

A U B = {x : x ϵ A or x ϵ B}

So, A U B = {2, 4, 6, 8, 10, 12},

here the common elements are not repeated.

Properties of (A U B)

- Commutative law holds true as (A U B) = (B U A)
- Associative law also holds true as (A U B) U {C} = {A} U (B U C)

Let A = {1, 2} B = {3, 4} and C = {5, 6}
A U B = {1, 2, 3, 4} and (A U B) U C = {1, 2, 3, 4, 5, 6}
B U C = {3, 4, 5, 6} and A U (B U C) = {1, 2, 3, 4, 5, 6}
Thus, the law holds true and is verified.

- A U ϕ = A (Law of identity element)
- Idempotent Law – A U A = A
- Law of the Universal set (**U**): (A U **U**) = **U**

Intersection of Sets

An intersection is the collection of all the elements that are **common** to all the sets under consideration. Let A = {2, 4, 6, 8} and B = {6, 8, 10, 12} then A ∩ B or "A intersection B" is given by:

"A intersection B" or A ∩ B = {6, 8}

Mathematically, A ∩ B = {x : x ϵ A **and** x ϵ B}

Properties of the Intersection – A ∩ B

The intersection of the sets has the following properties:

- Commutative law – **A ∩ B = B∩ A**
- Associative law – (**A ∩ B) ∩ C = A ∩ (B∩ C)**
- ϕ ∩ A = ϕ
- U ∩ A = A
- A∩ A = A; Idempotent law.
- Distributive law – **A ∩ (BU C) =** (**A ∩ B) U (A ∩ C)**

Difference of Sets

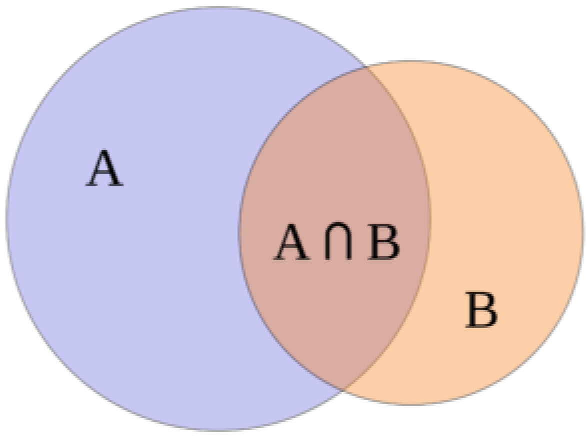
Difference of two sets A and B is the set of elements which are present in A but not in B. It is denoted as A-B.

Let A = {3, 4, 8, 9, 11, 12 } and B = {1 , 2 , 3 , 4 , 5 }. Find A – B and B – A.

Solution: We can say that A – B = {8, 9, 11, 12} as these elements belong to A but not to B

B – A = {1,2,5} as these elements belong to B but not to A.

Let, A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8} then A – B = {1, 3, 5} and B – A = {8}. The sets (A – B), (B – A) and (A ∩ B) are **mutually disjoint sets**; it means that there is NO element common to any of the three sets and the intersection of any of the two or all the three sets will result in a null or void or empty set.



Complement of Sets

If U represents the Universal set and any set A is the subset of A then the complement of set A (represented as A') will contain ALL the elements which belong to the Universal set U but NOT to set A.

Mathematically, $(A)'$ **= U – A**

Alternatively, the complement of a set A, A' is the difference between the universal set U and the set A.

Properties of Complement Sets

- A U A' = U
- A ∩ A' = ϕ
- De Morgan's Law – $(A \cup B)'$ = $(A)'$ ∩ $(B)'$ OR $(A \cap B)'$ = $(A)'$ U $(B)'$
- Law of double complementation: $(A)'$ = A
- $\emptyset'$ = U
- U' = ϕ

Solved Examples For You

Question 1: Let A = {1, 3, 5, 7} B = {5, 7, 9, 11} and C = {1, 3, 5, 7, 9, 11, 13} prove that:

(A∩ B) U (A ∩ C) = A ∩ (BU C)

Answer : B U C = {1, 3, 5, 7, 9, 11, 13}
A ∩ (B U C) = {1, 3, 5, 7}
Hence, A ∩ B = {5, 7}
A ∩ C = {1, 3, 5, 7}
(A∩ B) U (A∩ C) = {1, 3, 5, 7} ... {Hence proved}

Question 2: If A = { 1, 2, 3, 4} and U = { 1, 2, 3, 4, 5, 6, 7, 8} then find A complement (A').

Solution :
A = { 1, 2, 3, 4} and Universal set = U = { 1, 2, 3, 4, 5, 6, 7, 8}

Complement of set A contains the elements present in universal set but not in set A.

Elements are 5, 6, 7, 8.

∴ A complement = A' = { 5, 6, 7, 8}.

Question 3: If A = { 1, 2, 3, 4, 5 } and U = N , then find A'.

Solution :
A = { 1, 2, 3, 4, 5 }

U = N

⇒ U = { 1, 2, 3, 4, 5, 6, 7, 8, 9,10,… }

A' = { 6, 7, 8, 9, 10, … }

Question 4:f A = { x | x is a multiple of 3, x ∉ N }. Find A'.

Solution :
As a convention, x ∉ N in the bracket indicates N is the universal set.

N = U = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,11, … }A = { x | x is a multiple of 3, x ∉ N }

A = { 3, 6, 9, 12, 15, … }

So, A’ = { 1, 2, 4, 5, 7, 8, 10,11, … }

C.U QUESTIONS AND ANSWERS

1. If A ={1,2,3,4,5,6,7,9,11,13,15} and B={2,4,6,8,10,12,14,16,18}, find (A-B)U(B-A) , (A-B) ∩ (B-A)

SOL: A-B ={1,3,5,7,9,11,13,15} and B-A={8,10,12,14,16,18}

So , (A-B)U(B-A) ={1,3,5,7,9,10,11,12,13,14,15,16,18}

And (A-B) ∩ (B-A) = ϕ

2. If A = {1,2,3,4} , B= {3,4,5} ,C ={1,4,5} Verify the following statement:
A-(BUC) =(A-B) ∩(A-C)

SOL: BUC ={1,3,4,5}
A-(BUC) ={2}
Again A-B ={1,2} and A-C ={2,3}
(A-B) ∩(A-C)={2}
A-(BUC) =(A-B) ∩(A-C)

3. If A = {x: x is an integer and 1<x<10} and B = {x: x is an integer multiple of 3 and 5 and x≤30} .Find AUB, A∩B**,** (A-B),(B-A)

   SOL: A= {2,3,4,5,6,7,8,9} and B= {3,5,6,9,10,12,15,18,20,21,24,25,27,30}

   AUB = {2,3,4,5,6,7,8,9,10,12,15,18,20,21,24,25,27,30}
   A∩B = {3,5,6,9}
   A-B = {2,4,7,8}
   B-A ={10,12,15,18,20,21,24,25,27,30}

5. If $U = \{1, 2, 3, 4, 5, 6\}$ be the universal set A, B, C, are three subsets of U where $A = \{1, 3, 4\}$ and $B \cup C = \{1, 3, 5, 6\}$. Find (i) $(A \cap B) \cup (A \cap C)$ and (ii) $(B \cup C)'$. [1999]

Solution (a) $\{1, 3\}$, (b) $\{2, 4\}$

6. (a) Given $A = \{2, 3, 8\}$, $B = \{6, 4, 3\}$, find $A \times B$.

(b) Find the power set of $\{1, 2, 3\}$.

Solution (a) $A \times B = \{(2, 6), (2, 4), (2, 3), (3, 6), (3, 4), (3, 3), (8, 6), (8, 4), (8, 3)\}$

(b) $\{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{3, 1\}, \{2, 3\}, \{1, 2, 3\}, \phi\}$ [2003]

7. Using set theory find the HCF of the numbers 12 and 15.

Solution Let A and B be the two sets, where A and B are the sets of factors of 12 and 15.

Then $A = \{1, 2, 3, 4, 6, 12\}$

$B = \{1, 3, 5, 15\}$, $A \cap B = \{1, 3\}$, The highest is 3.

Then the HCF of 12 and 15 is 3.

8. Using set operators, find the HCF of 21, 45, and 105.

Solution 3 (Similar to question 7). [2005]

9. If $A = \{1, 2, 7\}$ and $B = \{3, 5, 7\}$ are the subsets of the universal set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then show that $(A \cup B)' = A' \cap B'$.

Solution $A' = \{3, 4, 5, 6, 8, 9, 10\}$

$B' = \{1, 2, 4, 6, 8, 9, 10\}$

$A' \cap B' = \{4, 6, 8, 9, 10\}$.

Again, $A \cup B = \{1, 2, 3, 5, 7\}$

$(A \cup B)' = \{4, 6, 8, 9, 10\} = A' \cap B'$. [2005]

10. If $A \cup B = \{a, b, c, d\}$, $A \cap B = \{b, c\}$, $A \cup C = \{a, b, c, d\}$, and $A \cap C = \{a, b\}$, then find A, B and C

Solution Let $U = (A \cup B) \cup (A \cup C) = A \cup (B \cup C) = \{a, b, c, d, f\}$

$A \cap B = \{b, c\} = \{b, c\} \subseteq A$

$A \cap C = \{a, b\} = \{a, b\} \subseteq A$

$\therefore A = \{a, b, c\}$.

Again, $A \cap B = \{b, c\}$, then $\{b, c\} \subseteq B$

$A \cup B = \{a, b, c, d\}$ $\therefore B = \{b, c, d\}$

and $A \cap C = \{a, b\}$ then $\{a, b\} \subseteq C$ again $A \cup C = \{a, b, c, f\}$ $\therefore C = \{a, b, f\}$. [2008]

11. If $A \cup B = \{p, q, r, f\}$, $A \cup C = \{q, r, s, f\}$ $A \cap B = \{q, r\}$, $A \cap C = \{q, s\}$, then find the set A, B, and C.

Solution $A = \{q, r, s\}$, $B = \{p, q, r\}$, $C = \{s, q, f\}$. [2009]

12. Find the power set of the set $\{2, 4, 6\}$.

Solution $\{\{2\}, \{4\}, \{6\}, \{2, 4\}, \{4, 6\}, \{6, 2\}, \{2, 4, 6\}, \phi\}$. [2010]

13. If $A = \{2, 3, 4, 5\}$, $B = \{1, 2, 3, 4\}$. Show that $B - A \neq A - B$.

Solution $A - B = \{5\}$ $B - A = \{1\}$ $\therefore A - B \neq B - A$. [2011]

14. If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15\}$

$B = \{2, 4, 6, 8, 10, 12, 14, 16\}$

Show that $A - B \neq B - A$.

Solution $A - B = \{1, 3, 5, 7, 9, 11, 13, 15\}$

$B - A = \{8, 10, 12, 14, 16\}$

$\therefore A - B \neq B - A$.

THE END