

# SURFACE TENSION

## LECTURE-2

## Introduction

It was seen during the study of vapour pressure that the molecules at the surface of a liquid experience a net inward pull because of the larger number of molecules towards the liquid side than towards the vapour side. There is a tendency on the part of surface molecules to go in the bulk of the liquid. The surface of liquid is therefore in some sort of tension and it tends to contract to the smallest possible area in order to have the minimum number of molecules at the surface. It is for this reason that the surface of a liquid is spherically curved, since the surface area is minimum for a given volume in the case of a sphere.

## Surface Energy

If the area of the surface is to be extended then one has to bring more molecules from the bulk of a liquid to its surface. This will require expenditure of some energy because work has to be done in bringing molecules from the bulk against the inward attractive forces. The amount of work done in increasing the area by unity is known as the *surface energy*. Larger the forces of attraction amongst the molecules of a liquid, larger the net inward pull, and thus, larger work will be involved in increasing the area. Hence, the surface energy can be used to define these net inward pulls.

## Surface Tension

It is customary to define these inward pulls in terms of the surface tension. The latter is defined as *the force acting along the surface of a liquid at right angle to any line of unit length*. The surface tension is equal, both numerically and dimensionally, to the surface energy. This can be proved as follows:

### Equivalency of Surface Tension and Surface Energy

Consider a line of unit length (say, 1 cm) anywhere on the surface of a liquid (Fig. 2.4.1).

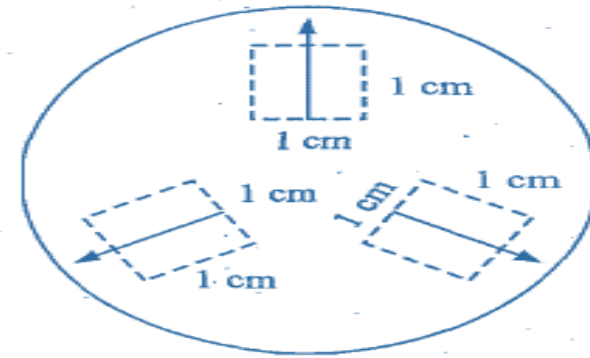


Fig. 2.4.1 Representation of surface tension

The surface area of the liquid can be increased by pulling this line perpendicularly by a force acting along the surface of the liquid. Let this line be extended by unity so as to enclose a unit area of the surface. Then

$$\text{Surface energy} = \frac{\text{amount of work done}}{\text{amount of area extended}} = \frac{\text{force} \times \text{distance}}{\text{area}}$$

Hence in CGS units, we have

$$\text{Surface energy} = \frac{\text{dyn} \times \text{cm}}{\text{cm}^2} = \frac{\text{dyn}}{\text{cm}} = \text{surface tension}$$

and in SI units, we have

$$\text{Surface energy} = \frac{\text{N} \times \text{m}}{\text{m}^2} = \text{N m}^{-1}$$

Obviously,  $1 \text{ dyn cm}^{-1} = (10^{-5} \text{ N}) (10^{-2} \text{ m})^{-1} = 10^{-3} \text{ N m}^{-1}$

The surface tension is represented by the symbol  $\gamma$ .

## Formation of Bubbles

The formation of a bubble is basically due to surface tension. It is obvious that the total pressure acting on a concave side must be larger than the pressure acting on the convex side. Therefore, the pressure inside a bubble must be larger

than the external pressure. If this excess of pressure is not balanced by any other force, then the bubble will not be stable and will collapse immediately. Since stable bubbles are formed, this excessive pressure must be balanced by certain forces. These forces are due to surface tension and are called the *cementing forces*.

Let a gas bubble in a liquid be cut into two hemispheres by an imaginary plane (Fig. 2.4.2). There will be a tendency for the two halves to be driven apart by a force equal to the inward pressure multiplied by the surface area of the hemispheres. This will be prevented by the external forces due to atmospheric pressure and by the cementing forces acting along the circumference of the circle. For a stable bubble, the total disruptive forces must be equal to the binding forces.

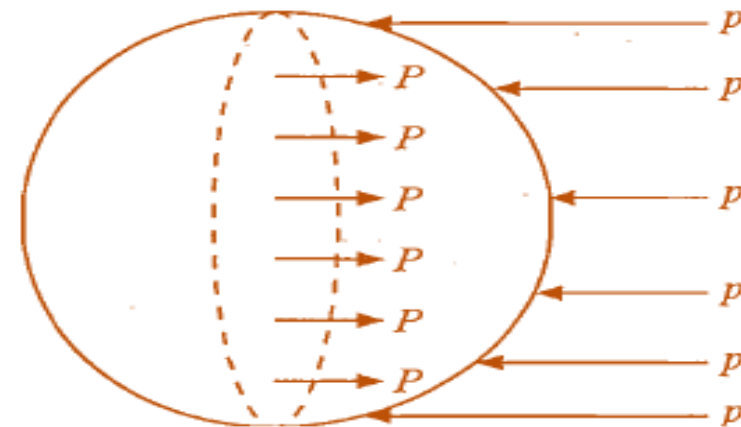


Fig. 2.4.2 Formation of a bubble

## Expression of Excessive Pressure

We have

$$\text{Disruptive forces} = P(\pi r^2)$$

$$\text{Binding forces} = p(\pi r^2) + \gamma(2\pi r)$$

Thus, for a stable bubble,

$$P(\pi r^2) = p(\pi r^2) + \gamma(2\pi r) \quad \text{i.e.} \quad \gamma(2\pi r) = (P - p) \pi r^2$$

or

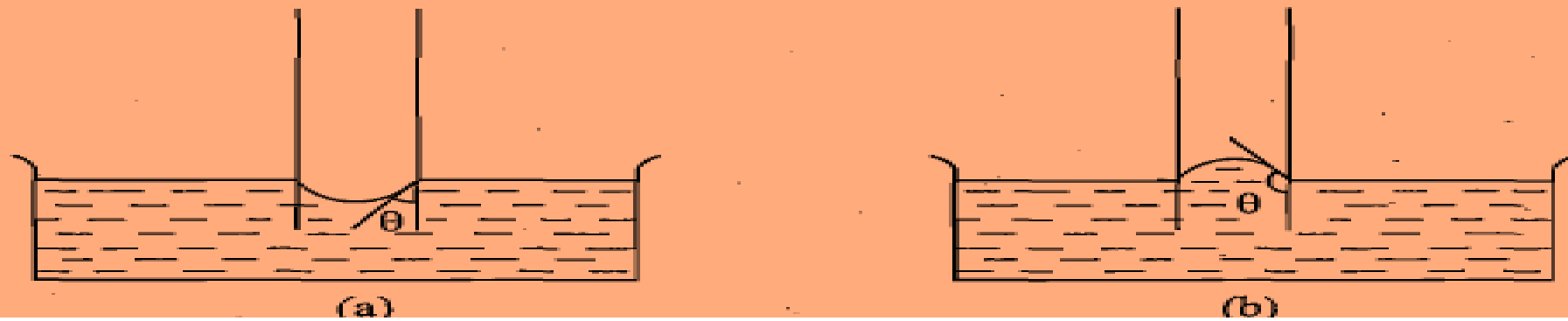
$$(P - p) = \frac{2\gamma}{r}$$

➤ The excessive pressure inside the spherical bubble is thus inversely related to its radius. From this, it follows that the formation and maintenance of smaller bubbles will need greater values of excess pressure than the larger ones. This is, in fact, in agreement with the daily experience of blowing a balloon. It is difficult to blow a balloon initially but becomes easier later on as the excess pressure to be generated inside the balloon is inversely related to its radius.

### Phenomenon of Capillary Action

When a capillary tube is dipped in a liquid, there occurs either a rise or a fall of liquid in the tube. This phenomenon is known as *capillary action* and is basically due to surface tension of the liquid. If the forces of attraction between the molecules of a liquid and those of the solid surface of the tube are greater than those existing amongst the molecules of the liquid, then the liquid has a tendency to spread on the solid surface and its meniscus in the tube is concave upwards. Such types of liquids are known as *wetting liquids* and they rise in the

capillary tube. The angle of contact, which is measured within the liquid from the side of the tube to the tangent drawn at the meniscus touching the surface of the tube, in this case, is less than  $90^\circ$ . This is shown in Fig. 2.4.3a. If the cohesive forces in the liquid are greater than the solid-liquid attraction forces, or if there occurs repulsion between the molecules of the liquid and those of the solid surface, the liquid detaches from the surface of the solid. The meniscus of such a liquid in the tube is convex upwards and its level falls within the tube. The angle of contact, in this case, is greater than  $90^\circ$  as shown in Fig. 2.4.3b.



**Fig. 2.4.3** The angle of contact of the meniscus (a) wetting liquid,  $< 90^\circ$  and (b) non-wetting liquid,  $> 90^\circ$

## Expression of Surface Tension

As mentioned above, the rise and fall of a liquid in a capillary tube is due to surface tension. Take, for example, the case of a wetting liquid. The surface tension forces act all around the capillary tube in the direction shown in Fig. 2.4.4. The liquid rises in the tube because of these upward forces. It continues to rise till the vertical component of the lifting force becomes equal to the weight of the liquid in the capillary tube. Thus,

$$\text{Lifting force} = (\gamma \cos \theta) (2\pi r_c)$$

$$\text{Weight of the liquid in the capillary tube} = \{(\pi r_c^2)h\} \rho g$$

At equilibrium, lifting force is equal to the downward force due to weight of the liquid in the capillary tube. Therefore,

$$(\gamma \cos \theta) (2\pi r_c) = \pi r_c^2 h \rho g$$

$$\text{or} \quad \gamma = \frac{1}{2} h \rho g \frac{r_c}{\cos \theta} \quad (2.4.1)$$

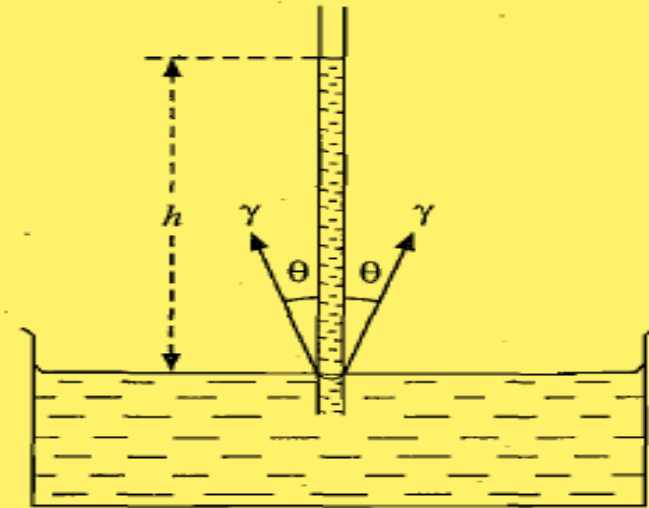


Fig. 2.4.4 Rise of a liquid in a capillary tube



For most of the wetting liquids,  $\theta$  is very very small and thus  $\cos \theta \approx 1$ . Therefore, the value of  $\gamma$  for such liquids is

$$\gamma = \frac{1}{2} h \rho g r_c \quad (2.4.2)$$

For non-wetting liquids, Eq. (2.4.1) is still applicable. Here  $h$  denotes depression of the liquid level.

## Measurement of Surface Tension

### Method Using Capillary Action

The phenomenon of capillary action can be used to determine surface tension of a liquid. However, to avoid the necessity of determining  $r$ , we use the relative method in which the value of  $h$  is also determined for a liquid whose surface tension is known. Thus

$$\gamma_1 = \frac{1}{2} h_1 \rho_1 g r, \quad \text{and} \quad \gamma_2 = \frac{1}{2} h_2 \rho_2 g r$$

$$\text{Hence, } \frac{\gamma_1}{\gamma_2} = \frac{h_1 \rho_1}{h_2 \rho_2} \quad \text{or} \quad \gamma_1 = \left( \frac{h_1 \rho_1}{h_2 \rho_2} \right) \gamma_2 \quad (2.4.3)$$

Hence, for known values of  $\gamma_2$ ,  $h_1$ ,  $\rho_1$ ,  $h_2$  and  $\rho_2$ , the value of  $\gamma_1$  can be calculated. The accuracy of this method can be further increased by the use of two capillary tubes of different radii as shown in Fig. 2.4.5. We have

$$\gamma = \frac{1}{2} h_1 \rho g r_1 \quad \text{or} \quad \frac{\gamma}{r_1} = \frac{1}{2} h_1 \rho g$$

$$\text{and} \quad \gamma = \frac{1}{2} h_2 \rho g r_2 \quad \text{or} \quad \frac{\gamma}{r_2} = \frac{1}{2} h_2 \rho g$$

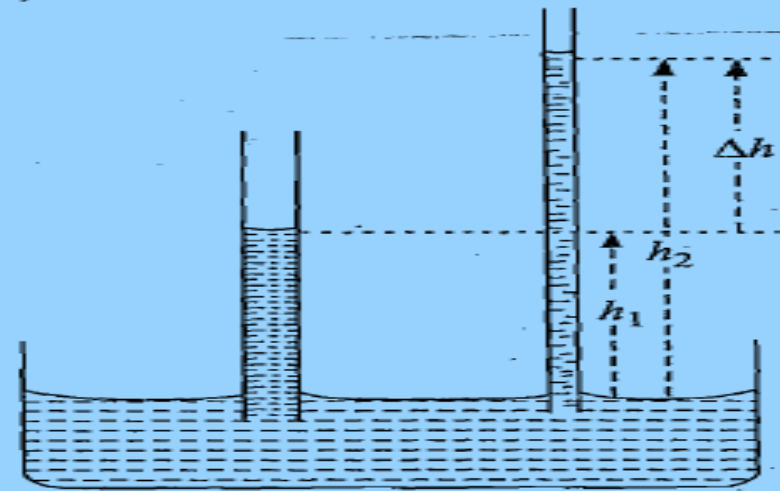
Subtracting, we have

$$\gamma \left( \frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{1}{2} \rho g (h_1 - h_2) = \frac{1}{2} \rho g \Delta h$$



$$\text{or } \gamma = \frac{\rho g \Delta h}{2 \left( \frac{1}{r_1} - \frac{1}{r_2} \right)}$$

**Fig. 2.4.5 Capillary rise method for measurement of surface tension**



If the same pair of capillaries, after proper cleaning, are dipped in two different liquids one after the other, we will have

$$\gamma_A = \frac{\rho_A g \Delta h_A}{2 \left( \frac{1}{r_1} - \frac{1}{r_2} \right)} \quad \text{and} \quad \gamma_B = \frac{\rho_B g \Delta h_B}{2 \left( \frac{1}{r_1} - \frac{1}{r_2} \right)}$$

Dividing, we get

$$\gamma_A = \frac{\rho_A \Delta h_A}{r_B \Delta h_B} \gamma_B \tag{2.4.4}$$

Thus, knowing the values of various terms on the right side of Eq. (2.4.4), the value of surface tension of the liquid A can be determined.

## A Laboratory Method

The other method which is commonly employed in laboratories is to use a *stalagmometer* (Fig. 2.4.6).



Fig. 2.4.6 The stalagmometer

A stalagmometer consists of a bulb attached to a fine capillary with a sharp edge. The liquid is allowed to fall drop by drop through the capillary tube. As the size of a liquid drop grows, its weight goes on increasing. It remains attached to the edge because of the forces of surface tension which act around the circumference of the tube and in the upward direction. When the downward force, due to weight of the liquid, becomes infinitesimally greater than the upward forces of surface tension, the drop detaches from the apparatus and falls down. Thus, the weight of the drop can be correlated with the surface tension of the liquid:

$$\text{Upward force} = (2\pi r) \gamma$$

$$\text{Downward force} = mg$$

Thus,  $mg = (2\pi r) \gamma$

In order to avoid measurement of  $r$ , we use the relative method in which the experiment is repeated with a liquid whose surface tension is known. Usually water is employed for this purpose. Thus,

$$m_1 g = 2\pi r \gamma_1 \quad \text{and} \quad m_w g = 2\pi r \gamma_w$$

Dividing and rearranging, we get

$$\gamma_1 = \left( \frac{m_1}{m_w} \right) \gamma_w \quad (2.4.5)$$

The values of  $m_1$  and  $m_w$  can be determined following either the drop weight method or the drop number method.